Peirce's relativization of the analytic vs. synthetic dichotomy

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Abstract: Kant introduced the (in)famous analytic-synthetic distinction in philosophy. Among other definitions, an analytic proposition is said to be a proposition whose predicate is "covertly contained" in its subject. C.S. Peirce repeatedly examined this distinction with critical interest, and recognized both the usefulness and the limitations of classifying all propositions into these two types. Long before Quine's attacks on this so-called dogma, he suggested the need to strongly relativize the dichotomy. The article is an attempt to list Peirce's major arguments against considering all propositions as either analytic or synthetic and against the general relevance of the dichotomy. Without pretensions of being exhaustive, the main arguments are: the fact that non-ampliative judgments may also produce an increase in knowledge; that the logic of relatives proves the distinction to be irrelevant; that the Existential Graphs also reshape the line between the analytic and the synthetic; that a proposition may be indifferently analytic or synthetic depending on what universe of discourse it refers to; that analytic truths are no more necessary than synthetic truths; and that the purely given on which synthetic propositions are based is a myth.

Keywords: Analytic; Synthetic; Peirce; Proposition; Judgment; Dogma (of empiricism).

Kant is known to have introduced the analytic-synthetic distinction in philosophy for clarification purposes[1]. While the *a priori* vs. *a posteriori* distinction is about the source or justification of

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[1] In this article, I will not open the particularly dense chapter of the analyticity of mathematics or of their being synthetic *a priori*, which notably includes the question of logicism. On the one hand, Peirce's position with regard to the synthetic *a priori* is not always very clear. On the other hand his criticism would rank him, alongside Bolzano, Frege, Carnap, Gödel, etc., among the overwhelming majority of philosophers, logicians and mathematicians who saw to some extant in the synthetic *a priori* an unnecessary fantasy of Kant's. This would not do justice to the originality of the Peircian approach.

our knowledge, the analytic-synthetic distinction is about the content of judgments or propositions[2]. Kant was the first philosopher to claim that the analytic-synthetic distinction should be «indispensable with regard to the critique of human understanding, and therefore deserves to be classical in it» (Kant, 2004: 22). However, the distinction has been famously "quined"[3] in the 20th century. An attentive reader of the *Critique of Pure Reason*, Peirce recognized both the usefulness and the limitations of this distinction, which he repeatedly examined with critical interest. A quick glimpse might lead one to believe that he maintained it[4]. However, Peirce brought a significant shift to it. In spite of assiduous study of Kant*,* he refused to admit in the end that propositions are divided into two and only two categories, analytic and synthetic. He went so far as to write that Kant's «distinction between the analytic and synthetic judgments […] is so utterly confused that it is difficult or impossible to do anything with it» (CP 5.176, 1903)[5]. Long before Quine's attacks on the dichotomy[6], Peirce suggested the need to strongly relativize this distinction. He alleged at least the following (non-ordered) grounds. First, an analytical judgment may produce an increase in information. As a consequence, it is not contrary to an "ampliative" judgment. Second, the distinction does not stand up to the method of the logic of relatives. Even more so is

[2]   By maintaining this distinction, Peirce remained much more Kantian than Frege. For Frege, the analytic and the synthetic concern not the content of the judgment but the legitimacy of the act of judging. He related the analytic-synthetic distinction to sources of knowledge. This means that the distinction refers to a type of evidence: rather than defining analytical judgment as a judgment in which the predicate is included in the subject, it is characterized as what can be proven in a purely logical way.

[3]   *Philosopher lexicon*, Daniel Dennett ed., 1987, APA edition: quine - v. « (1) To deny resolutely the existence or importance of something real or significant».

[4]   Although thoroughly examined, it was also S.-J. Shin's conclusion: «Peirce himself eventually reached an accurate definition very close to Kant's analytic/synthetic distinction» (Shin, 1997: 2). Otte (2006) more or less agrees.

[5]   The full quotation reads: «Now Kant's conception of the nature of necessary reasoning is clearly shown by the logic of relations to be utterly mistaken, and his distinction between analytic and synthetic judgments, which he otherwise and better terms explicatory (*erläuternde*) and ampliative (*erweiternde*) judgments, which is based on that conception, is so utterly confused that it is difficult or impossible to do anything with it».

[6]   I don't claim that Quine was the first to "relativize", to say the least, the dichotomy. For instance, one year before the paper on the two dogmas, Morton White published «The Analytic and the Synthetic: an Untenable Dualism», in a book on John Dewey (White, 1950).

it undermined by the Existential Graphs, a chapter that will not be opened in the present article, however important it is. Thirdly, the distinction corresponds to a difference in the universe of discourse, not to an essential difference in propositions. Moreover, analytical truths are no more necessary than synthetic truths, which are for their part also based on leading principles. One can add that for Peirce there is no such thing as the purely empirical (the outside of Quine's "web of knowledge"), because the absolutely given such as a first intuition is a myth. The following pages are an attempt to give a coherent frame to these various reasons for relativizing the classical dichotomy of the analytic and the synthetic according to Peirce.

1. *Of tautologies and definitions: are they analytic?*

Quine's critique begins with the observation that there are in fact two classes of statements usually referred to as analytical. Some are analytical because they are logically true, and some because they are true by virtue of the mere definition of the terms used. "No unmarried man is married" falls into the first group and is formalized: $\neg(\exists x)(Mx \wedge \neg Mx)$. "No bachelor is married", on the other hand, has a perfectly consistent logical form: $\neg(\exists x)(Bx \wedge Mx)$. What Quine rejects is the idea that one can find a universal criterion distinguishing this last type of propositions from synthetic propositions of the same logical form. Rather than denying that being analytic is a real characteristic of certain propositions, he therefore confines it to the meaning of logical constants. Beyond this, the distinction between the analytic and the synthetic has no precise and definitive border.

Peirce admitted at some point the restriction of the analytic to what is absolutely beyond any empirical verification, that is, to purely logical laws and principles. He thus acknowledged a slight twist in Kant's analytic vs. synthetic divide: the analytic judgment is narrowed towards the pure *a priori*, «so as to make it not merely needless but impossible to test one by experience» (W1: 274). This excludes a possibility evoked in the § 2 of the *Prolegomena*: «all analytical propositions are still *a priori* judgments even if their concepts are empirical» (Kant, 2004: 17). These judgments named analytic

by Kant correspond to the "essential predication" among scholastic philosophers[7].

For Kant too, analytical propositions have a very restricted use: they serve above all to clarify concepts – a concern which obviously could not leave a pragmatist indifferent. But seeing them as a tool for semantic clarification, Kant did not really question the logical analytic to which Peirce reduced analytical propositions[8].

Actually, Kant gave several definitions of the analytic[9]. Traditionally, one distinguishes four of them in the *Critique of Pure Reason*: one in terms of containment (strongly defended by Anderson (2015) as its most fundamental definition against Proops and others), a second in terms of identity ("7+5 = 12", i.e. substitutability in extensional contexts), a third one in terms of the explanatory-ampliative contrast[10], and a fourth one with the notion of «cognizability in accordance with the principle of contradiction»[11]. As to which one is most essential to Kant's concept of the analytic, critics disagree. Peirce generally seems to consider that the distinction can be conflated with that between «analytic, or explicatory, and synthetic, or ampliative, propositions» (CP 4.43, 1894).

The idea of containment is thus explained in the introduction to the *Critique of Pure Reason*: «Either the predicate *B* belongs to the subject *A* as something that is (covertly) contained in this concept *A*; or *B* lies entirely outside the concept *A*, though to be sure it stands in connection with it. In the first case I call the judgment

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[7]   Cf. the definition in Baldwin's *Dictionary*: «Essential predication: in which the predicate is wholly contained in the essence of the subject. It is, therefore, in Kant's sense, an analytical judgment» (CP 2.361, 1900). For another historical explanation: «Kant seems to have been the first to throw any light upon the subject [*of modalities*]. To the old distinction between logical and real possibility and necessity he applied two new pairs of terms, analytic and synthetic, and subjective and objective» (CP 2.385, 1900).

[8]   According to De Jong (2010: 259) «Kant simply does not apply the analytic-synthetic distinction to (general) logic at all».

[9]   One should add the traditional definition of logical truth as "derivability from the principle of contradiction".

[10]   Also in the *Prolegomena*: analytic judgements are «merely explicative and add nothing to the content of the cognition»; synthetic judgments are «ampliative and augment the given cognition» (Kant, 2004: 16).

[11]   Also in the opening of the *Prolegomena*: «All analytic judgments rest entirely on the principle of contradiction and are by their nature *a priori* cognitions». Synthetic judgments «can by no means arise solely from the principle of analysis, namely the principle of contradiction; they demand yet a completely different principle» (Kant, 2004: 17).

analytic, in the second synthetic» (Kant, 1998: 141). It was famously exemplified by the two propositions, "all bodies are extended" and "all bodies are heavy". However, the formalization of the two propositions shows that the two examples taken by Kant have exactly the same logical form, namely: $(\forall x)(Fx \rightarrow Gx)$. Only the construction of rigorous semantics could perhaps make it possible to decide about any statement whether it is analytic or synthetic, since their logical syntax is not sufficient to distinguish them. This is indeed the gist of Quine's argument against the possibility of a precise, universal and immutable criterion for the dichotomy.

As for the idea of explicative and ampliative force, Kant writes: «Analytic judgments say nothing in the predicate except what was actually thought in the concept of the subject, though not so clearly nor with the same consciousness» (Kant, 2004: 62). As noticed by De Jong (2010: 244): «Kant is not saying that analytic judgments have no content or are cognitively empty; he is saying that they do not augment the content of the cognition. And that is something else». An analytic judgment is not empty, but it makes a (confused) concept distinct by explicating one of its marks as a predicate. It is also explained in Kant's *Logic*, § 36: «To every x, to which the conception of body (a + b) belongs, extension (b) also belongs; this is an example of an analytic proposition. To every x, to which the conception of body (a + b) belongs, attraction (c) too belongs; this is an example of a synthetic one» (Kant, 1819: 156).

Peirce set out to pose and examine this problem of the supposed lack of content of analytical propositions from his 1868 articles at least to the theory of Existential Graphs. In his *Grand Logic* he quoted *in extenso* the famous passage from the introduction of the *Critique of Pure Reason* previously mentioned which coins the concepts of analytic and synthetic propositions and exposes the examples of extended bodies and heavy bodies. Peirce commented:

Like much of Kant's thought this is acute and rests on a solid basis, too; and yet is seriously inaccurate. The first criticism to be made upon it is, that it confuses together a question of psychology with a question of logic, and that most disadvantageously; for on the question of psychology, there is hardly any room for anybody to maintain Kant right. Kant reasons as if, in our thoughts, we made logical definitions of things we reason about! (CP 4.85, 1894)

Peirce clearly stated that the dichotomy is "seriously inaccurate".

The first remark to be made is that propositions that are supposed to be analytic are not necessarily so for the subject who thinks them. Psychologically speaking, they are not definitions, because the subject who thinks them learns something from them. That is to say, if logical analysis can ultimately reduce them to identity, the propositions in question have other epistemic virtues of clarification and explanation that cannot be ignored in the name of logic. It is a psychological absurdity to believe that what is implicitly contained in a thought is thought all the same, albeit more or less unconsciously. Otherwise, it would suffice to examine very carefully the premises of number theory to discover for example the truth of Fermat's theorems, Peirce insisted (CP 4.52, 1894).

The degree of knowledge of the subject must be taken into account. Since «synthetic propositions are those whose predicate is not in the subject as it is represented» (R 741, c. 1867), the distinction between analytic and synthetic propositions depends on our knowledge and representations: if we know (perceive, infer, etc.) that the predicate belongs to the subject, then, the proposition is analytic. Logical analysis can show that an apparently synthetic proposition is not: «whatever is absolutely universal is devoid of all content or determination, for all determination is by negation. The problem, therefore, is not how universal propositions can be synthetical, but how universal propositions appearing to be synthetical can be evolved by thought alone from the purely indeterminate» (W2: 143n). «The difference between a logical inference which gives rise to a verbal proposition, and a "material" sequence which gives rise to a "real proposition"» (W6: 273) must be questioned. In particular, mathematical propositions are not analytical in the sense of Kant (their predicate is not included in the subject), but they are not synthetic either: they do not affirm facts, but relate to "hypothetical states of things", which account for the necessity of mathematics (CP 4.232, 1902).

2. *Analytic, synthetic, and extensive propositions*

The puzzlement that led to the introduction of the analytic vs. synthetic dichotomy may be encapsulated in the following remark. Bodies, extended entities and heavy entities are coextensive, but

in a different way: whereas bodies and heavy things could have referred to two distinct classes, one cannot conceive that what is denoted by "body" did not strictly coincide with what is denoted by "extended thing". From this perspective, the problem of the analytic and synthetic is but the problem of coextension. «Admit that there are no coextensive notions (and there never are *known* to be) and the system of propositions is reduced to great simplicity» (W1: 337).

The problems appear when a proposition involves a subject and a predicate of the same extension, because it conveys no information. Information should be here understood as a product of logical extension and comprehension. Peirce called "information" «the sum of synthetical propositions in which the symbol is subject or predicate» (W2: 83), and which results from the breadth and depth of the terms that compose them. An analytic proposition is a proposition, that is, the predication "S is P" attributed to a subject, in which the predicate does not teach anything about the subject, or does not affirm anything about it. There are two cases in which a proposition is not affirmative. «This is when, in the very same act by which we learn that S is P, we also learn that P was covertly contained in the previous depth of S, and that consequently S was a part of the previous breadth of P. In this case, P gains in extensive distinctness and S in comprehensive distinctness» (W2: 84) What Peirce describes here without naming it is the case of the Kantian analytic proposition, and "covertly contained" is the exact expression of Kant.

An example of the other case is: «a boundary is both within and without what it bounds» (W2: 83). In reality, the boundary has no breadth. Since «a contradiction consists in giving to contradictory terms some breadth in common» (*ibidem*), the contradiction stated here is only apparent. Peirce seems to be playing on an ambiguity of "breadth", which from a semantic sense slides to literally spatial. One can infer from the preceding development that the two types of analytic judgment are thus the explicitation of a predicate "covertly contained" in the subject on the one hand, and the predication to a subject without extension on the other. The discourse on nothing would thus be a case of analytical judgement as non-synthetic: it does not bring knowledge about some object, since nothing true or false can be asserted about it. Whether we call it analytical or non-analytical, it is therefore the same category of propositions (i.e.,

a kind of simile-propositions) that includes the always true and the
never true (nor false).

Peirce's theory is fundamental to understanding the supposed in-
clusive relationship between subject and predicate, which has been
the subject of many criticisms[12]. Peirce did not stop there. For it is
still necessary to analyze the "covertly contained" of the analytic.
This refers to the definitional use of the proposition – although in a
non-Kantian way, since Kant distinguished precisely analytic defini-
tions from synthetic definitions in his *Logic*.

> A defining proposition has a meaning. It is not, therefore, a merely identical
> proposition, but there is a difference between the definition and the definite.
> According to the received doctrine, this difference consists wholly in the fact
> that the definition is distinct, while the definitum is confused. But I think there
> is another difference. The definitum implies the character of being designated
> by a word, while the definition, previously to the formation of the word, does
> not. Thus, the definitum exceeds the definition in depth, although only verbally.
> (W2: 85-86)

Peirce drew an important consequence from this: the defining
proposition affirms that whatever a certain name is applied to is
supposed to have such and such characters, but not necessarily vice-
versa. When I say that a mouse is a small rodent having a pointed
snout and small rounded ears, I do not imply that all rodents with
the same characters are mice, although they «certainly might be so
called» (W2: 86). Hence, reasoning from definitum to definition is
a deduction, but reasoning from definition to definitum is a certain
kind of hypothesis.

So far, the previous analysis does not suffice to dismiss the ana-
lytic-synthetic dichotomy, but shows that Peirce was keener on us-

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[12] Among many others: «What does it mean to say that a certain concept is in-
cluded in another? We can understand clearly enough the meaning of inclusion in its
usual, i.e., spatial connotation, as when we say this room includes a shelf of books, or the
English alphabet includes twenty-six letters, but in application to concepts, the sense of
the term remains singularly obscure. It suggests, following the spatial analogy, that the
subject term is somehow logically wider than the predicate term so that in the necessary
judgment, the "logically wider" term represents the subject which contains or includes
the "logically smaller" predicate term. But this "logical spacing", if we may so call it, is a
very misleading analogy. In what sense is the subject term in the proposition $7 + 5 = 12$
logically wider than the predicate term? And in the proposition "red is a color", it is the
predicate that appears to be more extensive in logical space than the subject» (Wild and
Coblitz, 1948: 653).

ing other tools (such as breadth, depth, information) which tend to soften the dichotomy. In particular, not all definitions, or rather not all uses of defining propositions, are deductive. However, in developing the pragmatist theory, Peirce came to place the clarification of conceptions through the examination of conceivable possible practical consequences at the heart of the philosophical enterprise. The definition considered as a stipulation, a precept for action, can hardly remain an analytical statement[13].

Pushed to its limits, the analysis of propositions in terms of information is devastating. Among other texts, the 10th Harvard Lecture gives a good sense of it as early as 1865 (W1: 272-286). After defining comprehension as accounted for by the meaning (or connotation) of a sign, and extension as accounted for by its applicability to objects (or denotation), the text explains the difference between extensive and comprehensive propositions. An extensive (resp. comprehensive) proposition states the relation between the extension (resp. comprehension) of two terms. For instance, "Black horse is contained *under* horse" is extensive (the "containment" refers to horses), and "Horse is contained *in* black horse" is comprehensive (the "containment" refers to meanings). Then it comes to the Kantian division of propositions: «In analytical judgments there is no denotation at all. In a synthetical judgment the subject is an object of denotation» (W1: 273). Indeed, an analytic proposition is such that both its subject and predicate are objects of connotation: for instance, in order to subsume "mortal" under

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[13]   Cf. CP 2.330, 1903: «The peculiarity of this definition – or rather this precept that is more serviceable than a definition – is that it tells you what the word lithium denotes by prescribing what you are to do in order to gain a perceptual acquaintance with the object of the word. Every subject of a proposition, unless it is either an Index (like the environment of the interlocutors, or something attracting attention in that environment, as the pointing finger of the speaker) or a Sub-index (like a proper name, personal pronoun or demonstrative) must be a Precept, or Symbol, not only describing to the Interpreter what is to be done, by him or others or both, in order to obtain an Index of an individual (whether a unit or a single set of units) of which the proposition is represented as meant to be true, but also assigning a designation to that individual, or, if it is a set, to each single unit of the set. Until a better designation is found, such a term may be called a Precept. Thus, the Subject of the proposition, "Whatever Spaniard there may be adores some woman" may best be regarded as, "Take any individual, *A*, in the universe, and then there will be some individual, *B*, in the universe, such that *A* and *B* in this order form a dyad of which what follows is true", the Predicate being "___ is either not a Spaniard or else adores a woman that is ___".»

"man" and say analytically that "Man is mortal", it is requisite that
only the significations be considered. But in a synthetic judgment,
the subject must be understood as applying to things. "All men
are mortals" is synthetic in this sense. At this point, Peirce made a
difference between two kinds of synthetic propositions, according
to whether the predicate is denotative or connotative. If connota-
tive, it may be called "synthetic intensive", and "extensive" if both
subject and predicate denote. Thus, there are not two but three
fundamental types of proposition: analytic, synthetic intensive, and
extensive. «There is no such thing as an analytic extensive proposi-
tion» (W1: 273). There cannot either be a connotative subject with
a denotative predicate, since it is a law that «Every symbol *denotes*
by *connoting*»: hence, the connotative term which determines the
other is necessarily a predicate. In an extensive proposition, some
information is gained, which corresponds to an extension of both
comprehension and extension.

Hence, since symbols are determined by their objects; and there are three
objects of symbols the connotative, denotative, informative; it follows that there
will be three kinds of propositions, such as alter the denotation, the informa-
tion, and the connotation of their terms respectively. But when information is
determined both connotation and denotation are determined; hence the three
kinds will be 1st Such as determine connotation, 2nd Such as determine de-
notation, 3rd Such as determine both denotation and connotation. (W1: 277)

There are, then, three kinds of propositions – a proposition be-
ing now conceived as something which *alters* the denotation, con-
notation or information of its terms. First, analytic or connotative
propositions, like "Logic is science". In it, the conception of logic
«embodies the *form*» of science, Peirce explained. «Here *logic* is
immediately determined as to connotation and science is *mediately*
determined in denotation» (W1: 277). The second kind of propo-
sitions is the class of extensive or denotative propositions, because
through them the predicate is immediately determined in denota-
tion. Peirce gives the example of "Unripe fruit is green", which de-
termines both unripe fruit to be green and green things to include
unripe fruit (W1: 278). The third kind is synthetic intensive, or in-
formative, propositions, for instance when «I describe the Russian
Plague by giving an example of it; I determine the example medi-
ately. That is when I find out the connotation of Russian Plague I

shall know more about this case but immediately I only determine the Russian Plague and that only in denotation» (*ibidem*). Peirce eventually mentioned a last case, as "*Homo* is man", when there is a gain of information in both terms. Strangely enough, this last case, which is dismissed as «only of grammatical import», could be interpreted both as the most informational, in Peirce's terminology, and the more analytic, because it is only linguistic. It shows the limits of the analytic/synthetic dichotomy in the frame of Peirce's theory.

Importantly, the three kinds of propositions do not correspond to different facts: there is no analytic fact as distinct from an empirical fact. The difference is only a semiotic question of being represented: «we see that the *denotative object* and the *connotative object* are in fact identical; and therefore an analytic, an intensive, and an extensive proposition may all represent the same fact and yet the mode in which they are obtained and the relation of the proposition to that fact are necessarily very different» (W1: 275).

Analyzing the proposition in terms of information, we can then say that, of the two Quinian examples, "No unmarried man is married" and "No single man is unmarried", the first is not a proposition and the second is not analytical. For "No unmarried man is unmarried" (or in Peirce's example of an identical proposition, "All red crows are red", W1: 247) does not contain the minimum information necessary to form a proposition[14]. As for the second statement, Peirce came to interpret the "covertly contained" as a call for a necessarily empirical explicitation. Only observational facts can make us understand that what is hard scratches the diamond, that triangles have not only three angles but three sides, and perhaps that the state of celibacy is incompatible with marriage. Of course, it is necessary to include internal, ideal observation, the experience of the intimate sense of consciousness, without limiting the empirical to some so-called empirically given. A proposition whose predicate is indistinctly wrapped in the subject is therefore a synthetic proposition, Peirce remarked in his *Grand Logic*:

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[14]   Many authors agree. One could mention Wild and Coblitz 's principle of "connotative diversity": As a sheer repetition of an unanalyzed concept, of a single term, a tautology is not a proposition, because it lacks the connotative diversity that is requisite between subject and predicate. «Any sentence whose constituent terms are not connotatively diverse is not a proposition» (1948: 655).

What Kant calls an explicative, or analytical, judgment is either no judgment at all, because void of content (to use his phrase), or else it sets forth distinctly in the predicate what was only indistinctly thought (that is, not actually thought at all) in the subject. In that case, it is really synthetic, and rests on experience; only the experience on which it rests is mere internal experience – experience of our own imaginations. Association by resemblance, and association by contiguity: all lies in that great distinction. (CP 2.451n, 1894)

3. *"Covertly contained": the role of observation in the logic of relatives*

A second argument against the division of propositions into analytic and synthetic comes from the logic of relatives. Peirce considered that the distinction stems from an outdated logic, and reflects the unsatisfactory state of logical studies in Kant's time. It is unfortunate that so many philosophers were «entrapped by Kant's view of the relation between his Analytic and Synthetic Judgments», when the study of the logic of relatives «would at once have exploded» such a view (CP 5.84, 1903). For «our ideas about the distinction between analytical and synthetical judgments is much modified by the logic of relatives, and by the logic of probable inference. An analytical proposition is a definition or a proposition deducible from definitions; a synthetical proposition is a proposition not analytical» (CP 6.595, 1893).

By redefining, as he did here in the *Monist*, the analytic no longer on the basis of propositions but of reasonings, Peirce was taking a decisive step. He echoed the conviction that illation is the fundamental logical relation, and the thesis «That Categorical and Hypothetical Propositions are one in essence», the title of a 1896 manuscript. For if a proposition links two parts by means of the verb "to be", an appropriate analysis of the copula tells us that this link also operates a transition from one idea to the other. Almost every categorical proposition «amounts to the assertion that something it describes may be otherwise described» (W4: 249). These two descriptions are, in the tradition, the subject and the predicate – which Peirce reinterpreted initially as symbol and ground (W2: 57). This is obvious when a proposition connects a subject to a predicate by a copula, but to take a less intuitive example: «The proposition, "If it lightens it will thunder", can be put into

the form, whatever is denoted by the term, "The state of things in which it lightens", is also denoted by the term, "The state of things in which it thunders", or, "The state of things which will be followed by thunder"» (W3: 85).

The subsumption of the subject under the predicate is also a transition from one to the other, so that hypothetical and categorical propositions belong in reality to one and the same category (cf. also W1: 337). Their formulation is equivalent: «Any fact which can be stated in the form of a hypothetical proposition can also be stated in the form of a categorical proposition» (W4: 247). «By thus identifying the relation expressed by the copula with that of illation, we identify the proposition with the inference, and the term with the proposition» (W4: 170). In short, the algebra of logic shows that it is indifferent to speak of subject, antecedent or premise, and predicate, consequent or conclusion. An inquiry into analytical propositions is therefore a questioning about the nature of analytical reasoning, which is deductive reasoning.

A similar twist, «so slight that their application remains almost exactly the same» (W1: 246), was given to the pair *a priori/ a posteriori*: in accordance with its historical sense prior to Kant, *a priori* and *a posteriori* refer to a reasoning from an antecedent to a consequent and vice-versa (W1: 245), or from cause to effect and vice-versa (W2: 106). The phrase «inferred *a posteriori*» thus means «determined from without the mind by something not previously present to it, being so determined their determinants or causes/reasons are not present to the mind and of course could not be reasoned from» (W1: 246). That is why «a same predicate may be analytic in stating a consequent and synthetic in stating an antecedent» (R 741, c.1867). It blurs the distinction between analytic and synthetic, which are but ways of describing the way an inference is conducted, or even perhaps parts or moments of the same inferential process.

In first approximation, therefore, the difference between analytic and synthetic propositions is that between a conclusion obtained immediately from a reasoning and a conclusion obtained by another type of inference, not mathematical. «By synthetical judgments he meant such as assert positive fact and are not mere affairs of arrangement; in short, judgments of the kind which synthetical reasoning produces, and which analytic reasoning cannot yield» (W3: 304). As for the analytic:

If the implied proposition be thought, it is thought in some cryptic sense, and it in no wise tells us how it is that inference is performed, to say that in such sense the conclusion is thought as soon as the premises are given. The distinction between analytical and synthetical judgments represents this conception of reasoning. The distinction may approximate to a just and valuable distinction; but it cannot be accepted as accurately defined. (CP 4.52, 1894)[15]

But this is only a first approximation, which cannot be taken for granted. A whole theory of the types of inferences and how they work remains to be proposed. However, despite their great differences, "deduction, or analytical reasoning" and "scientific or synthetical, reasoning" share an important point in common: «Deduction is really a matter of perception and of experimentation, just as induction and hypothetic inference are; only, the perception and experimentation are concerned with imaginary objects instead of with real ones» (CP 6.595, 1893). In other words: «analytical reasoning depends upon associations of similarity, synthetical reasoning upon associations of contiguity» for deduction rests on associations in our consciousness, and ampliative reasoning on cosmic laws in the world. As a result, observational errors are always possible, so that analytic reasoning is not necessarily correct. Moreover, the deduction may only be probable. It pushes a wedge between the analytic and the necessary. Moreover, even the *a priori*, whose criterion in Kant's work is the universal and the necessary, is not free from error: «I cannot see that they afford the slightest reason for thinking that such propositions are ever absolutely universal, exact, or necessary in their truth. On the contrary, the principles of probable inference show this to be impossible» (*ibidem*).

In the 1890s, when Peirce wrote this text, the ideas on which this comparison is based were not justified by the Existential Graphs but by the logic of relatives: «The logic of relatives, which

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[15]   The rest of the text (R 408: 147ff.) is not published in the *Collected Papers*. It would appear that it deals with the law of association. With respect to this quote, Shin (1997: 2-3) notes that: «Quite surprisingly, a striking phrase in this quotation, "The distinction may approximate to a just and valuable distinction", in which Peirce endorses Kant's analytic/synthetic distinction, has not received any attention. Instead, only Peirce's criticism of Kant's ignorance of polyadic logic has been highlighted in the philosophy of logic and mathematics». This seems to me justified by the overwhelming weight of "cannot be accepted" versus "may approximate".

justifies these assertions, shows accordingly that deductive reasoning is really quite different from what it was supposed by Kant to be; and this explains how it is that he and others have taken various mathematical propositions to be synthetical which in their ideal sense, as propositions of pure mathematics, are in truth only analytical» (CP 6.595, 1893). The Existential Graphs would only confirm this intuition of an analytic character of all deductive reasoning, but in the sense of analytic that encompasses experimentation and observation.

If the logic of relatives blurs the boundary between analytic and synthetic, it is not only because the analytic implies observation. It is also because, correlatively, this observation brings to light new things that were not foreseen in the premises. It is not only a matter of making appear what was covertly contained in the premise, it is also a matter of making appear radically new things.

> Since Kant, especially, it has been customary to say that deduction only elicits what was implicitly thought in the premisses; and the famous distinction of analytical and synthetical judgments is based upon that notion. But the logic of relatives shows that this is not the case in any other sense than one which reduces it to an empty form of words. Matter entirely foreign to the premisses may appear in the conclusion. (CP 3.641, 1900)

An example of this is the possibility of mathematically deducing from an extremely simple definition a proposition of the greatest complexity. It is what Peirce called his «first real discovery about mathematical procedure» (NEM 4: 49, 1902): the Corollarial and the Theorematic are two different kinds of necessary reasonings. In the latter, the deduction requires an experiment in imagination on the image of the premiss in order to reach a conclusion, while in a corollary the conclusion is deduced directly without any construction (Hintikka, 1980). The analytic vs. synthetic dichotomy is much less significant than that between corollaries and theorems. This aspect of Peirce's works in logic, which has been most studied, will not be developed in the present paper, for it would require a study of its own[16].

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[16]   The situation is clearly summed up by Stjernfelt (2007: 108): «It is unclear whether Kant's concept of analytic judgment covers corollarial deductions only, or both corollarial and theoretical deductions, simply because Kant did not have a corresponding distinction at his disposal. Hintikka argues that Kant's concept of analytic inferences covers corollarial reasoning only, so that theorematical inferences must be classified as

Suffice it to recall that Kant was obliged to qualify mathematics as synthetic in order to

> provide for the fact that an indefinitely complicated proposition, very far from obvious, may often be deduced by mathematical reasoning, or necessary deduction, by the logic of relatives, from a definition of the utmost simplicity, without assuming any hypothesis whatever (indeed, such assumption could only render the proposition deduced simpler); and this may contain many notions not explicit in the definition. (CP 2.361, 1900)

The alternative is the following: the kind of hypothetical proposition of the type "If a man is a rational animal, then X", with X a highly complex consequence, is either analytical, and then Kant's definition is not at all adequate, or it is synthetic, but as a necessary consequence it does not correspond to the Kantian sense either.

It could be objected that Kant wanted to admit some synthetic that is nevertheless necessary and universal, namely the form of the synthetic *a priori*, in order to encompass cases of this kind. The previous hypothetical proposition ("If a man is a rational animal, then X") is a synthetic judgment although not an accidental predication. One could suspect that Peirce was prevented from admitting it because he conflated accidental with synthetic. Was not Peirce deceived by his thinking in scholastic terms of essential predication instead of analytic judgments?

Some proposed to replace the analytic/synthetic dichotomy by the distinction of essential and accidental attributes indeed. For example, Horace Joseph, an Oxford professor rather hostile to symbolic and mathematical logic, stated:

> No judgment is analytic in the sense of asserting of anything in the predicate what in the subject-concept we have already realized or indicated it to be. What Kant has really done is to distinguish those judgments in which the predicate is part of the definition of the subject from those in which it is not. The distinction we may mark by the antithesis *essential* and *accidental* […]. But the opposition of *analytic* and *synthetic* is misleading, since that insight into the nature of a subject which definition expresses, though it may be called an analysis, is also an apprehension of the connexion of elements in an unity, and the necessity of this connexion cannot be derived from the law of Contradiction. (Joseph, 1906: 211)

synthetic – corresponding to Peirce's idea of diagrammatical reasoning as covering the synthetic a priori domain». Levy (1997) also claims that the analytic/synthetic partition follows the division between corollarial and theorematic reasoning.

However, to cling to an Aristotelian usage which presupposes a whole metaphysics of substantial essences and natural kinds which does not belong to modern science would be hazardous. As a matter of fact, Peirce did not confuse essential predication and analytic propositions, since he carefully distinguished between them in an entry in Baldwin's *Dictionary*. "Per se" is said in different senses:

These five senses are, then: (1) that a substance exists *per se* and not *per accidens*; (2) that an analytical proposition is true *per se*, or formally, and not as matter of fact; (3) that any character which a thing necessarily assumes by virtue of existing, belongs to it *per se*, and not *secundum quid*; (4) that which a thing causes of itself it does *per se*, and not *per aliud*; and (5) that which any abstraction, *qua* that which it is, is, does, or suffers, is *per se* and not *secundum quid*. (CP 6.385, 1900)

There are various ways of making a predication *per se* according to essence, definition, presence, cause or abstraction. These rather cryptic examples show at least that Peirce did not identify what is predicated *per se* with the analytic, which only corresponds to the second case. That is to say that the analytic is reserved for verbal definition, or what is implied by the meaning of a word. In this sense, we could say that if Peirce rejected the dichotomy of the analytic and the synthetic, it is partly because it is much too simplistic with regard to the subtlety of scholastic distinctions. As Peirce said elsewhere: «Propositions were further distinguished into propositions *per se* and propositions *per accidens*. But this was a complicated doctrine, which Kant very conveniently replaced by the distinction between analytic, or explicatory, and synthetic, or ampliative, propositions» (CP 4.43, 1894). To accept the analytic-synthetic distinction is convenient, but might be a form of intellectual laziness and oversimplification.

4. The «difficult problem of the essential nature of a Proposition»

The "essential nature of a Proposition", as Peirce called it, lies largely in the fact that it bears on the possible. This is what distinguishes it from judgments and assertions. In the jungle of uses and the "ontological slums" (as Quine put it), statement, asser-

tion, judgment, proposition and sentence deserve conceptual clarification. German philosophy traditionally contrasts the judgment (*Urteil*) with the proposition (*Satz*) as the *actus mentis*, by which the spirit unites or separates (typically, a predicate and a subject) with the *enunciatio*, i.e. the meaning of this judgment for oneself or for others (see for example Wolff's *Logica*). In this framework, the proposition refers to meaning only insofar as it is linguistic, whereas it is the judgment, though certainly psychological in nature, which proceeds to the logical linking of the elements.

Peirce sometimes did not bother with such distinctions: «it is to be understood that proposition, judgment, and belief are logically equivalent (though in other respects different) and since the proposition is more tangible it is more convenient, in formal logic, to speak of that» (W4: 402). Nevertheless, Peirce was most often hostile to the "German" use:

> The writer, like most English logicians, invariably uses the word *proposition*, not as the Germans define their equivalent, *Satz*, as the language-expression of a judgment (*Urtheil*), but as that which is related to any assertion, whether mental and self-addressed or outwardly expressed, just as any possibility is related to its actualization. The difficulty of the, at best, difficult problem of the essential nature of a Proposition has been increased, for the Germans, by their *Urtheil,* confounding, under one designation, the mental *assertion* with the *assertible*. (EP 2: 339)

How then to account for the fact that Peirce also sometimes took up the German distinction and spoke of judgment as a «relation of ideas» and of «proposition or expression of a judgment» (W3: 90)? Far from being a contradiction, it is rather a result of his semiotic approach. For the proposition "expresses" the judgment not in the sense that it verbalizes it, but in the sense that it is a sign of it: «There must be a sign which signifies that one thing is the sign of another. A sign which does this is called a proposition; the corresponding thought a judgment. In the proposition then there is reference to two signs one of which is represented as standing for whatever the other stands for» (W3: 95). The proposition is thus both the objective content of the judgment and its external expression, two poles that seemed totally opposed in the Bolzanian conception of the proposition in itself. This is because it is a sign, which has real objectivity but can take different forms of instantiation. The impor-

tant matter is that some form of the sign is necessary in language and «necessary in thought which is equivalent to a language» (W3: 96).

"Proposition" is first and foremost the logical term («any statements, or as we say in logic, any *propositions*», W8: 252). It is «the consecrated term of logic, which Appuleius, in the second century of our era, already speaks of as familiar» (W8: 64). Peirce sometimes relied on an ancient treatise, Aldrich's *Rudiments* of 1690, which he considered «guiltless of any fanciful thinking», to point out the following essential distinctions (W8: 86): the logical operations of the mind may be divided into simple apprehension, judgment and reasoning[17]. Judgment is «recognizing two ideas to be bound together in a fact». Or again: a judgment is «the inward act that we seem to perform when we first acquire» a habit-belief (W4: 43). «The representation to ourselves that we have a specified habit of this kind is called a *judgment*» (W4: 164). Through the three operations of the mind, three products are obtained: terms, propositions and syllogisms.

The distinctive feature of a proposition is to be a symbol determining not only a quality but also an object: propositions are «symbols which also independently determine their *objects* by means of other term or terms, and thus, expressing their own objective validity, become capable of truth or falsehood» (W2: 57). Peirce wrote in his «New List of Categories»:

And it is remarkable that, among all the definitions of the proposition, for example, as the *ratio indicativa,* as the subsumption of an object under a concept, as the expression of the relation of two concepts, and as the indication of the mutable ground of appearance, there is, perhaps, not one in which the conception of reference to an object or correlate is not the important one. (W2: 57)

This is what every proposition says, but it is also a characteristic phenomenon of interpretation and semiotics in general: «connected with any representation of an object there is another representation – an independent representation of the same object», that is, «a proposition or judgment is an essential part of the representation of the object» (W3: 64). A proposition always refers both to its object and to another sign or interpretant, but rather implicitly: «The proposition is wanting in the reference to an interpreting represen-

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[17]   In W6: 272-3, he mentioned among the three things which logic considers, «the concept, the judgment and the sequence».

tation but not in the explicit reference to its object» (W3: 64). The reference to an interpretant is more essential to an argument.

Consequently, a proposition is always about some kind of possibility. Neither an inner discourse nor a linguistic statement, it is simply not asserted as such. It is a content waiting for actualization:

Let us distinguish between the proposition and the assertion of that proposition. We will grant, if you please, that the proposition itself merely represents an image with a label or pointer attached to it. But to assert that proposition is to make oneself responsible for it, without any definite forfeit, it is true, but with a forfeit no smaller for being unnamed. (CP 5.543, 1902)

An unasserted proposition says nothing about the actual world. «Now an object can always be imagined, or at least supposed, which shall reunite any two descriptions that are not absolutely contradictory, so that a proposition that merely says that among supposable objects there is one of a given description to which another given description is applicable might as well be left unsaid» (W4: 249). In other words, synthetic propositions tell us nothing if they do not relate to a real object, because they only express possibilities. To say of something which is not logically impossible that it is possible, is to say nothing at all. The only limit to this possibility is the "absolute" contradiction. By forcing the contrast somewhat, one could say that while for Wittgenstein every analytic proposition is devoid of meaning and content, for Peirce it is the synthetic proposition to be so devoid, because it only states what is possible; but everything is possible, except the "absolute contradiction". Certainly, for Peirce, a tautology too is meaningless: «nonsensical forms are so readily made in categoricals that the usage of language has taken them up and attached meanings to them. "What I am telling you is true", and "A man is a man", are frequently heard, although these are, in the strictest sense, nonsensical» (CP 2.352, 1896)[18]. But while the

[18]   Amongst the authors for whom analytical propositions are not nonsensical, Peirce mentioned Lotze, who vindicated the necessity of the analytical categorical proposition thus: «Lotze finds that the meaning of the analytical judgment is illogical, since it identifies contraries. However, the meaning of this meaning is justified by its not meaning to mean that the terms are identical, but only that the objects denoted by those terms are identical. The analytic proposition is, therefore, admissible, because it is practically meant to mean a particular proposition, that is, one in which the predicate is asserted of all the particulars» (CP 2.387).

Tractatus sees tautology and contradiction as parallel, Peirce found the contradiction much more informative. A proposition which involves contradiction, that is which implies two contradictory things, «does imply, or mean, something. A self-contradictory proposition is not meaningless; it means too much» (CP 2.352). For it informs us about an impossibility, whereas a tautology only says that nothing is impossible in the connection it establishes.

The only true propositions are basically all universal tautological and all particular non-contradictory propositions. For «particular propositions necessarily imply the existence, real or logical, of their terms» (W6: 273). The false is the assertion of a contradictory existence or of an empirical universal proposition, because one would always find a (at least possible) counter-example to it. «An unlimited universe would comprise the whole realm of the logically possible. In such a universe, every universal proposition, not tautologous, is false; every particular proposition, not absurd, is true» (W4: 450-1). Why should any empirical law, any synthetic generality, be systematically disproved? Because if their contrary is logically possible, then it is real in the order of the possible. All general "synthetic truths" are false. An empirical law that would always be true, i.e. whose contrary would not be true in any possible world, would be a logical law. In short, a logical law is only a possibility never denied by experience. Even if Peirce did not say explicitly so, it is a way of relativizing the distinction between the logical and the empirical: logical laws have no essence other than that of agreeing with all possibilities.

It is finally because he understood the possible as logical possibility that Peirce refused the synthetic character of mathematical propositions. Kant had a formal understanding of the possible, in the sense of the forms of experience. As Stang (2011: 462) reads:

> If we analyze compatibility with the forms of experience in terms of logical compatibility with the forms of experience, then our knowledge of what is formally possible, given the forms of experience, would be analytic, rather than synthetic. On this supposition, we would be able to derive the theorems of geometry from the axioms by non-synthetic deductive methods, but Kant thinks this is impossible, because we must employ the irreducibly synthetic method of construction in pure intuition.

For Peirce, compatibility with the forms of experience is not the

sense of the possible to which the propositions essentially refer: it is a logical understanding of the possible, to which would be added a conception of real *possibilia* borrowed from Scotist metaphysics. In fact, Peirce proposed two conceptions of the possible for the propositions (CP 2.347): an epistemic possibility linked to our state of knowledge ("ignorantial, or negative"), and a properly logical possibility ("positive", when one can make a disjunction of the type "either A, or B, or C, or D, etc. is true", whose aggregate is the universe of possibility).

The relevant distinction that divides the unlimited universe into two, and passes within the class of synthetic propositions, is the true and the false. In the first category, tautologies and particular non-contradictory propositions; in the second, universal propositions and contradictions.

5. *Analytic or synthetic in a universe of discourse*

As we have seen, the fundamental difference is not between categorical and hypothetical propositions, but between universal and particular: one can never reduce "All men are mortal" (i.e. "It is false that some man is not mortal") to "Some man is not mortal". «There is an essential difference between asserting that something exists & saying that something doesn't exist» (W4: 496). Universal quantification renders the propositions «equivalent to hypotheticals, and these amount to denying the existence of something», while by existential quantification the propositions «affirm the existence of something and thus amount to denying that one thing follows from another» (W4: 254).

The real dichotomy among propositions is therefore not between analytical and synthetic. It is as follows: «1, Universal, affirming leading-principles, asserting non-existence, or 2, Particular, denying leading-principles, asserting existence» (W4: 477). The propositions never do more than affirm or deny a leading principle, i.e. state that one is or is not entitled to take a certain inference as a habit of conduct.

All universal propositions are "leading principles", i.e. principles that must be followed in order to make inference-habits. Does this mean that Peirce considered them all as logical truths, or tautological?

No doubt a distinction must be made between a leading principle in an unlimited universe, which is equivalent to a logical law, and a leading principle in the physical world for example (or in any other universe of discourse), which may consist of an empirical regularity allowing inferences to be made. For we rarely refer to the entire universe of the possible. Our language is part of a «limited universe of discourse» (the «universe of a proposition» as De Morgan called it, before talking of «the whole universe of thought, or a conceivably separate portion of it» (De Morgan, 1850)), whether it be the physical possible, historical existence, or the world of fiction, for example. As Peirce explained in the Note A of his 1883 *Studies in Logic*, the propositions of these universes (or of the "universes of marks") have particular functions, because contrary to what happens in the logical universe, the distinction of propositions in extension and comprehension is real: the width and depth of the subjects and predicates refer to real beings, not to pure possibilities. It is only in the logical universe that the absence of possession of a character necessarily implies the possession of the opposite character (that everything that is not white is non-white, for example) (W4: 451).

Consequently, the dichotomy of propositions into analytic and synthetic can only be thought of in terms of a certain universe of discourse. The synthetic propositions have a meaning and a truth-value because they relate to a particular universe (otherwise they are always false), analytic propositions relate to the unlimited universe and, as they are always true there, have no meaning. «For analytical propositions, though affirmative, cannot, as analytical, assert the real existence of anything» (CP 4.44, 1894). Peirce therefore explained in the *Grand Logic* that the essential question is "what we are talking about":

> If we are saying that some imaginable kind of thing does or does not occur in the real world, or even in any well-established world of fiction (as when we ask whether Hamlet was mad or not), then the proposition is synthetic. But when we are merely saying that such and such a verbal combination does or does not represent anything that can find a place in any self-consistent supposition, then, we are either talking nonsense, as when we say, "A woolly horse would be a horse", or else, we are, as Kant says, expressing a result of inward experimentation and observation, as when I say, "Probability essentially involves the supposition that certain general conditions are fulfilled many times and that in the long run a specific circumstance accompanies them in some definite proportion of the occurrences". (CP 4.43, 1894)

Rather than two kinds of propositions, it is therefore preferable to distinguish three kinds: propositions asserting something true or false in a limited universe; verbal combinations (which are not true propositions) asserting pure possibilities (expressing non-contradictory relations) that are obvious (and therefore meaningless); and propositions asserting possibilities that are not obvious, i.e. whose predicate is not seen at the moment the subject is asserted, «that subject having hitherto been obscurely apprehended» (*ibidem*). These latter propositions are logical identities, but provide an epistemic gain because the identity they assert is not obvious. The trichotomy mentioned in the quotation, which in Kant's case corresponds more or less to the synthetic *a posteriori*, the analytic *a priori* and the synthetic *a priori*, must therefore be reinterpreted as the discourse indexed to a (synthetic) reality, the nonsense (not contradictory but on the contrary devoid of meaning because it only expresses the absence of contradiction) and the (analytical) explanatory.

It is because a proposition concerns the possible that it is necessary not only to delimit a universe of discourse but also to accompany it with an indexical. For example, the proposition "All men are mortal" refers to «the countless objects in the universe, past and future», and asserts that they will never have the character of an «immortal humanity» (W4: 403). «Using possibility in this sense, the universe of a proposition may be defined as a series of possibilities to which the proposition refers but whose limits cannot be described in general terms but can only be indicated in some other way» (*ibidem*). «Every proposition has three elements. 1st an indication of the universe to which it relates, 2nd its general terms, 3d the connection of its terms» (W4: 402). The universe of the proposition means that every proposition «relates to something which can only be pointed out or designated but cannot be specified in general terms». The desirable object to which the proposition refers is related to «a variety of possibilities, often an infinite variety» (W4: 402) (e.g. the variety of possible uses of a door if it says "Do not open" on it, or the variety of changes in Hamlet's consciousness if I am talking about the irresolution of Shakespeare's character). «It is an important theorem of logic that no proposition whatever can be completely and fully expressed in general terms alone» (W4: 249), i.e. by a general sign conventionally referring to a general idea; one must furthermore refer to an object existing at the present time; this

restriction to a domain is an essential part of the meaning, which a general description cannot make. One needs «finger-pointings» (W4: 249-250). Thus, «the proposition itself simply represents an image with a label or pointer associated with it» (CP 5.543, 1902). «Such a purely demonstrative sign is a necessary appendage to a proposition, to show what world of objects, or as the logicians say, what "universe of discourse" it has in view» (W4: 250).

Once the universe of discourse is defined, one can then assign a truth-value to the various particular and universal propositions, distinguishing between true ones corresponding to the universe of discourse and false ones that refer to unrealized possibilities in the universe in question. A proposition is true if it correctly represents a state of affairs[19].

> A *state of things* is an abstract constituent part of reality, of such a nature that a proposition is needed to represent it. There is but one *individual,* or completely determinate, state of things, namely, the all of reality. A *fact* is so highly a prescissively abstract state of things, that it can be wholly represented in a simple proposition, and the term "simple", here, has no absolute meaning, but is merely a comparative expression. (EP 2: 378, 1906)

It can then happen that a proposition is true of all individuals in a domain without being universal across all domains, whatever the universe in which it is interpreted. In other words, a true universal proposition is not necessarily universally valid.

6. *No proposition is necessary, therefore analytic*

Although the present paper does not aim at tackling the matter of the *a priori-a posteriori* dichotomy, which should be considered separately from the analytic-synthetic dichotomy, it is not totally indifferent to remark that, apart from the controversial synthetic *a priori,* most synthetic propositions are *a posteriori* and all analytic propositions are *a priori* (except for Kripkeans). An argument for relativizing the dichotomy of analytic vs. synthetic propositions starts from the observation that, if there are absolutely analytic propositions, they must be *a priori.* Kant claimed that universality and necessity

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[19]   Cf. W6: 20: «Truth being the conformity of a statement with fact».

are criteria of the *a priori.* Hence, by *modus tollens*, if no proposition is universal and necessary, then no proposition is *a priori;* therefore, no proposition is analytic. It is an indirect but rigorous demonstration: one cannot admit both that there are analytic propositions and that no proposition is universal and necessary.

Peirce sometimes very clearly stated that he disagreed that there are universal propositions: «Following the greatest students of the theory of cognition, I am disinclined to admit any proposition as absolutely necessary» (W6: 406). To clarify absolute necessity, Peirce quoted a Dutch logician, Franciscus Burgersdicius: «The absolutely necessary is that whose negation implies contradiction» (W2: 112) Hence, Peirce tended to doubt that there may exist propositions whose negation implies contradiction. It means that no proposition would be contradictory, not even "Not A is A". It may seem very unlikely.

> Reasoning, according to the doctrine of that work [Kant's first *Critique*], is regulated entirely by the principle of contradiction, which is the principle of analytical thought. The one law of demonstrative reasoning is that nothing must be said in the conclusion which is not implied in the premises, that is, nothing must be said in the conclusion, not actually thought in the premises, though not so clearly and consciously. (CP 4.52)

But it was Kant's logic. Peirce's position relies on a more modern logic, and is very much defendable. For "Not A is A" may be wrong, but not very wrong. Peirce thought about possible intermediary cases, like boundaries, or gradations. In particular, in a few manuscripts devoted to the nature of the proposition written when he worked with his students in logic at the Johns Hopkins University, he discussed the principle of excluded middle: «though a proposition be false it may have a certain value if it is not *very* false, – and indeed wherever continuity comes in, and here alone the mathematical logic is fully developed, no real proposition is exactly true, – so that the question is *how* false a proposition is» (W4: 490). The principle of excluded third is not false, but it falsifies any proposition referring to continuity, because it «cares not how little a statement errs so long as it is not exact truth» (W4: 493).

Although his position on this matter probably changed and evolved a lot through time, and was a search in progress rather than a body of definite claims, Peirce arguably «denies that even the axioms

of geometry and the laws of logic are absolutely necessary, universal, or exact» (Wilson, 2015: 209). He indeed claimed that absolute universality, absolute exactitude and absolute necessity cannot be attributed to propositions about objects of ordinary knowledge. Mathematical propositions cannot be more certain than objects of perception but in a very weak sense: «The operations of perception and of experimentation are subject to error, and therefore it is only in a Pickwickian sense that mathematical reasoning can be said to be perfectly certain. It is so only under the condition that no error creeps into it» (CP 6.595, 1893). To be sure, as Haack (1979) rightly insisted, fallibilism and necessity are not incompatible, and the necessity of logical and mathematical truths are no bar to the extension of fallibilism to logic and mathematics. But even the reality of absolutely necessary propositions is debatable.

Peirce's overall take was of course inspired by Kant. But it could also be argued that his position comes from a Leibnizian interpretation of the *a priori* (Chevalier, 2013). Whereas necessity was the mark of the *a priori* for Kant, it was self-evidence which was supposed to be its criterion prior to him: for Leibniz, what is *a priori* true, although not always necessary, is seen by the light of reason or demonstrated by the principles of identity and contradiction. Leibniz admitted that some *a priori* truths are contingent, because they are about facts. With Peirce as well, the *a priori* is no longer necessary, but is self-evident again, when presented in diagrams.

Despite this drift from Kantian orthodoxy, it remains that Kant's criticism was Peirce's horizon for his critical common-sensism: whatever is universally true is involved in the conditions of experience. But although it referred for Kant to the truth of mathematical and metaphysical propositions as well as propositions of pure physics, which he dubbed synthetic *a priori,* Peirce extended it to any universal proposition. That is why he repeatedly (over a period of at least thirty years) claimed that Kant's question should not have been "How are synthetical judgments *a priori* possible?" but "How are universal propositions relating to experience to be justified?", that is the possibility of synthetical judgments in general (e.g. W1: 248, CP 4.92). «How is it that a man can observe one fact and straightway pronounce judgment concerning another different fact not involved in the first?» (W3: 304). But as previously shown, not even the consequent of explicative inferences is always

"involved" in their premises, no more than synthetic judgments are statements of pure observation. If this question is truly «the lock upon the door of philosophy» (W2: 268), it is not only because it should lead to a satisfactory statement of the principle of induction. It is more profoundly because it reveals how the Kantian synthetic *a priori* was holding the role of an untenable intermediary between the logical analytic and the empirical synthetic. Peirce made it collapse onto the synthetic in general *and* in the same time get closer to the analytic, because it depends on a structure of experience which is a structure of rationality. For Peirce, all knowledge has an empirical origin, provided it is understood as the logical framework of experience, in particular a logic of induction, which is by no means different from the logic of general rationality. It provides a conciliation of Kant and Mill, as defended in Chevalier (2016), and blurs the distinction of the analytic and the synthetic.

The foregoing arguments may convey the feeling that a few impressionistic remarks are insufficient to justify a rival theory to that of Kant. And it is true that in many texts, Peirce took the analytic/synthetic dichotomy for granted. But if this is the case, it is only at the price of a profound refoundation of the distinction, on bases that end up drifting quite far from the initial Kantian inspiration. In overlooking the important criticism of the myth of the given and in making only a quick allusion to the Existential Graphs, the present article has somehow presented the path that lead from Peirce's first suspicions against the idea of a purely synthetic, correlative of a purely empirical intuition, to the new regime of the theorematic deduction in the diagrams. All these aspects converge towards a weakening of the dichotomy, which pragmatically culminated in C.I. Lewis' 1923 «A Pragmatic Conception of the A Priori» and perhaps even more in *Mind and the World Order*, and which would not fail to inspire Quine. The history of this pragmatic filiation remains to be written.

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