Sets, Separation, and Frames

*Giovanni Battista Ratti**

Abstract

The paper is devoted to analyzing, by using basic notions of set theory, the problem of the identification of law and the notion of a frame of meanings attributable to legal texts. The main results of the inquiry carried out in the paper are: 1) that only a neutral, non-evaluative, definition of law can do from a legal theoretical viewpoint; 2) that the concept of law should not be seen as referring to a unitary set of norms, but rather as referring to a plurality of sets and of results of operations on sets.

Keywords: Set Theory. Legal Positivism. Legal Interpretation. Frame of Meanings.

1. Introduction

In several works, I have used elementary notions of set theory to try and elucidate some fundamental problems of jurisprudence[1]. What I shall do here is to examine, again by means of a set theory approach, two basic and crucial issues for legal philosophy: the analysis of the relations between law and morality (both regarded as sets of norms), and the notion of an interpretive frame, on the footsteps of the analyses provided by Hans Kelsen and Riccardo Guastini. In so doing, I shall proceed as follows.

First, I will briefly go over some basic notions of set theory.

Second, I will try to demonstrate that, once the relations between law and morality are examined from both an extensional point of view and an intensional one,

* Tarello Institute for Legal Philosophy, Department of Law, University of Genoa, Via Balbi 30/18, 16126, Genova, Italy, gbratti@unige.it. Previous versions of portions of this paper have been presented at St. Hilda's College, University of Oxford, Oxford (UK), on June 5, 2013, and at the "XXXIV Jornadas Nacionales de Filosofía Jurídica y Social", Asociación Argentina de Filosofía del Derecho, Universidad de Buenos Aires, Buenos Aires (Argentina), on November 12, 2020.

1 Ratti 2013: ch. 12; 2019a; 2019b.

it is possible to conclude that only a strictly positivistic, non-evaluative, definition of law can be justified from a legal theoretical perspective.

Finally, I shall unpack Kelsen's and Guastini's notions of an interpretive frame from a set theory stance, showing that the concept of law is better understood as referring not to a unitary set of norms, but rather to an array of different sets, which must be carefully distinguished.

2. Set Theory Basics

Let me briefly introduce the definitions of the basic set-theory concepts[2].

The term 'set' denotes a collection of objects or individuals of any kind, regarding which it is possible to determine, by means of some definitional criteria, whether a certain object or individual belong to this collection or not. The objects or individuals which belong to a set are called 'elements' of the set. When a set has other sets as its elements, it is usually called a 'set of sets' or a 'family of sets'.

It is crucial for our present purposes to notice that a certain set can be identified extensionally, by enumerating its elements one by one, but it can also be identified intensionally, by pointing to the criteria which the objects must match to belong to it. This second way of identifying sets turns unavoidable when one deals with sets that, for empirical or conceptual reasons, are not susceptible of an extensional identification.

The extensional determination of a certain set is usually represented by the enumeration of all the elements which compose it between braces. For instance, if we want to identify the set α, composed by the members of *The Beatles*, we shall write {John, Paul, George, Ringo}.

By contrast, intensional identification can be expressed by stating that a certain property *F* determines a set, and, more precisely, the set of the objects or individuals which are characterized by the property *F*. Therefore, a certain object or individual *a* can be regarded as a member of a certain set, built up on the basis of property *F*, if, and only if, it, he or she possesses such a property[3]. For instance, "musician that played in a British Band with 28 UK top 10 hits in the sixties" could be one of the properties leading to the intensional identification of the set of *The Beatles*.

Two sets may be considered as identical if, and only if, they have the same elements. But, of course, two identical sets can be liable to two or more different in-

[2] See Berto 2007: 145 ff.; Lemmon 1965: appendix B; Quine 1961, 1963, 1974: 235 ff.; Russell 1918; Suppes 1960. The bases of such a theory are to be found in the works of Cantor, on which see Dauben 1990. See also Russell 2010: 67 ff.

[3] This apparently intuitive and innocuous principle leads to the well-known Russell's paradox, on which see at least Russell 2010: 80-81 and 101 ff.

tensional identifications. The set of the members of *The Beatles* can be intensionally identified by means of the above-mentioned property, but it can also be identified by means of the property "musician that played in the most famous rock band ever".

The extensional identification of a certain set can always be traced back to its intensional identification, but not vice versa. Indeed, the extensional identification of a certain set is often impossible, both for conceptual reasons, whenever the set to be identified is infinite (e.g. the set of natural numbers), and empirical reasons, whenever the set to be identified, though finite, is not liable of being completely known, due to the limited epistemic faculties of human beings (e.g. the set of all the living animals). This leads to two significant consequences.

First, it is possible to know a set which is not liable of being extensionally identified. However, such knowledge can never be extensionally complete, in so far as it will never bear upon all the elements of the set. In such cases, we can only obtain knowledge deriving from the intensional characterization of the set at hand and, if any, from a recursive method of identification of each element[4].

Second, it can be observed that, from an epistemic perspective, two different intensional presentations of co-extensional (viz. identical) sets may be not equivalent. For instance, I may know the extension of the songs written by Bruce Springsteen without knowing that the set of his songs may also be presented as 'the tunes of the Boss'.

A fundamental concept of set theory is 'inclusion'. Inclusion is the relation between two sets α and β, such that every element of α is also an element of β. One should distinguish *improper inclusion*, which admits the possible identity of both sets, from *proper inclusion*, which excludes it.

In other words, a set α *improperly* includes another set β, when the latter is a subset of the former, i.e. all the elements of β belong to α, which may (but need not) contain other elements. By contrast, a set α *properly* includes another set β when the latter is a subset of the former, which, in turn, need contain other elements (since, as has been affirmed, both sets cannot be identical).

It is important not to confuse inclusion with *membership*[5]. The former is a relation between sets (and only between sets); the latter is a relation between a set and its elements. The confusion often stems from two well-known circumstances: (*a*) inclusion is defined in terms of membership of elements to sets, so that, once inclusion is detected, the membership of an element to a certain (included) subset implies its membership to another (including) set; (*b*) it is possible that the elements of a set consist of other sets, so that the latter sets belong to the former set, but are not, despite appearances, included in such a set.

Another frequent confusion is the one between a set having only one member

[4] Russell 2010: 113.

[5] On the historical roots of such a confusion, Russell 2010: 79.

and its single member. On such a topic, Russell states that a set «consisting of one member is not identical with that one member» (id., 67). He gives the example of the expression "Satellite of the Earth" which denotes a set which, by chance, has only one member (i.e., the Moon), but could have many more. The phrase "Satellite of the Earth" – says Russell – is not altered in its meaning if other satellites of the Earth are discovered and is not devoid of meaning for those who do not know that, as a matter of fact, the Earth indeed has a satellite. By contrast, «Statements about the Moon […] are meaningless except to those who are aware of the Moon» (id., 67).

We can carry out several logical operations on sets, the main ones being the following: (1) *union*, (2) *intersection*, (3) *difference*, and (4) *complementation*. Let us briefly analyze them in this order.

(1) Given two sets α e β, their *union* consists in determining the set of all the objects which are members of the set α *or* of the set β. The union of the sets, say, α = {1, 2} and β = {2, 3} is, very simply, {1, 2, 3}.
(2) Given two sets α and β, their *intersection* consists in determining the set of all the objects which are members of both α and β. The intersection of the above mentioned sets is {2}.
(3) Given two sets α and β, we call *difference* the set of all the objects which belong to a set, but not to the other set. Unlike union and intersection, difference is not commutative: the order of the sets changes the outcome. The difference between α and β is the set of elements which belong to α, but not to β – in our example: {1}. By contrast, the difference between β and α is the set of the objects belonging to β, but not to α – in our example: {3}.
(4) Given a certain set α, we can carry out its *complementation*, which consists in determining the complementary class of α, which is composed of all the objects which do not belong to α. This class is usually represented by the notation α'[6].

Union, intersection, and complementation allow one to identify some logical relations among sets which are analogous to De Morgan's laws for propositional calculus[7]: the complementation of the intersection of sets α and β equates to the union of the complements of α and β. Analogously, the complementation of the union of α and β equates to the intersection of the complements of α and β.

[6] Complementation presupposes the previous identification of the *universe of discourse* within which it is carried out. This operation brings about a partition in the universe of discourse. Cf. Suppes 1960: 83 ff. Without this previous operation, complementation would be indeterminate, since the complementary class of a certain class a would be made of all the infinite classes belonging to any universe of discourse.

[7] As is known, for the laws of De Morgan, the negation of the conjunction of two propositions equates to the disjunction with both original propositions denied, and the negation of the disjunction of two propositions equates to the conjunction of both original propositions denied.

Another law of set theory states that, if α set a is included in β, then the complement of β is included in the complement of α. A banal example: if the set of Italians in included in that of the Europeans, then the set of the non-Europeans is included in that of non-Italians.

A notion of great relevance for set theory – and for philosophical analysis in general – is that of an 'empty set'. Such a set is determined on the grounds of criteria which are satisfied by no object and, as a consequence, contains no element. It is usually determined by stating unmatchable conditions, such as predicating an object of lacking identity with itself.

As a matter of course, all the empty classes are identical, since they have the same extension (i.e. no element), though their intensions may be different: take, for instance, the class of unicorns and that of half-gods. A special case of an empty set is given by the set identified by intersecting a certain set and its complement. A surprising property of the empty set is that it is included in any other set[8].

The complement of the empty set is the universal set, i.e. the set to which all the objects belong[9]. It is easy to see that the union of a set with its complement brings about the universal set. It is also easy to understand that the universal set contains any set[10].

With these basic instruments of set theory, we are now equipped to analyze the issues of the separation of law and morals and the notion of an interpretive frame.

3. Unsetting Connections: The Separation Thesis Revisited

The thesis of the separation of law and morals (henceforth, ST) – epitomized by the famous John Austin's maxim "The existence of law is one thing; its merit or demerit is another" – is one of the most controversial theses in legal theory. The interpretations of this thesis are so numerous and so diverging, that it is not always easy to understand which version of such a thesis is referred to, when it is advocated or questioned.

Personally, following a suggestion by Ross (1988: 148-150), I am keen on regarding ST as an eminently epistemic or methodological thesis, which bears on the knowledge of law, and not on law itself[11]. Since I've argued in favor of such a

[8] This is a principle of set theory that corresponds to the principle *ex falso quodlibet* of the propositional calculus.

[9] This set has received several interpretations: some understand it as the class of all the objects that belong to some set; others regard it as the class of all the objects. On the different notions of the universal set, and their philosophical relevance, see Alchourrón 1987. One of the few contemporary defenses of the second reading is found in Quine 1980: ch. 5, especially note 10.

[10] This is a set theory transposition of the propositional principle *verum sequitur a quodlibet*.

[11] Against this reading of ST, Green 2008: 1038 argues: "The separability thesis is not a meth-

reading of ST in other papers[12], I will not elaborate on such a reading here.

In this paper, I rather want to examine a "substantial" or "ontological" version of ST, according to which law is not necessarily connected to morality[13]. Framed in these terms, ST is, obviously, but the negation of the thesis of the necessary connection of law and morals (henceforth, CT)[14]. To fully understand the scope of ST, it is worthwhile to first examine CT[15].

In doing so, I will use as the main starting point for my analysis a well-known work by Leslie Green (2008), who carefully examines the notion of 'connection' between law and morals, and, consequently, surveys different interpretations of ST, understood as a series of "destructive arguments against the necessary connection thesis" (id., 1039).

Green holds that ST is false: according to him, several necessary connections between law and morals actually exist. Green's analysis is clarifying on many scores, but there are other points which can be fruitfully clarified by the set theory approach I have been presenting.

The first necessary connection of law and morals which Green spots is the following (2008: 1047): "Morality has objects, and some of those objects are necessarily law's objects". Some lines after, with a quotation from Kelsen, Green affirms that law and morals bear on the same object: namely, (social) relations between human beings. Some other lines on, Green (2008: 1048) adds that "there is a necessary relation between the scope of law and morality". These three sentences, when approached from a set theory stance, seem to affirm quite different tenets.

The first sentence seems to bear upon propositional contents of (legal and moral) rules, the second sentence bears upon the "regulative function" of law and morality, and the third sentence deals with the "topic" (the universe of discourse, as it were) regulated by such families of normative systems.

---

odological claim. It bears only on the object-level domain-that is, on laws and legal system". Green 2008:1039-1040 distinguishes ST from the sources thesis and from the social thesis: according to the *social thesis* «law must be grounded in social facts, and any non-factual criteria for the existence and content of law must likewise be grounded in such facts», whereas the *sources thesis* affirms that "the existence and content of law depends on its sources and not on its merits".

12 Ferrer, Ratti 2012a. See also Ferrer, Ratti 2010; 2012b.

13 According to Green 2008: 1041, ST is "the contention that there are no necessary connections between law and morality". Understood in this way, according to Gardner 2001: 223, ST is absurd: "Apparently legal positivists believe [that] there is no necessary connection between law and morality. This thesis is absurd and no legal philosopher of note has ever endorsed it as it stands. After all, there is a necessary connection between law and morality if law and morality are necessarily *alike* in any way. And of course they are. If nothing else, they are necessarily alike in both necessarily comprising some valid norms".

14 Obviously, CT, in turn, is but the negation of ST: this is so, since both theses are regarded as mutually exclusive and jointly exhaustive of the possible relations of law and morals. However, since ST is historically posterior to CT, and for many positivists, is a great progress towards a more scientific jurisprudence, I shall assume CT as the "primitive" thesis.

15 Cf. Green 2008: 1039.

With the first sentence, the thesis which Green seems to affirm is that law and morality (i.e. all the legal systems and all the moral systems) bear upon the same propositional contents (independently from the normative regulation that such systems assign to them). Of course, on this reading, such a thesis is exaggerated and, above all, false, as one could easily show by surveying legal and moral systems in force. What can be held is that, *usually*, there is an intersection of the set of propositional contents regulated by legal norms and the set of morally regulated contents. Obviously, there is nothing necessary in this connection. One could also hold that the set of the socially relevant propositional contents is determined by the union of the legally and morally legal contents, so that there is a sort of connection of these elements. But also in this case, one cannot see where the *necessary character* of this connection lies. Finally, one could hold that the set of the contents of a system is included in the set of the contents of the other. This is very much debatable, for there are many contents which are eminently legal, and others that are mainly moral, so that the relation would be one of intersection, and not of inclusion. At any rate, even if this were true, there would be no necessity at play either.

With the second sentence, Green refers to a more abstract level of analysis: to wit, to the presumptively common function of law and morals, that is guiding human conduct. This thesis is trivially true, by assuming the current definitions of 'law' and 'morals'. However, it sheds light neither on the regulation of normative contents nor on the identification of these contents. That law and morals have the same function guarantees neither that they regulate the same conducts, nor that the determination of such conducts receives the same intensional characterization[16], nor that the normative qualification of such conducts is the same. And it guarantees even less that the techniques used to guide conducts in the legal and the moral domains are the same: quite the opposite, law and morality are usually distinguished by virtue of the fact that law, unlike morality, provides the institutionalization of the procedures of sanctioning. As a consequence, one cannot see why such aspects allows one to affirm the existence of a necessary connection of law and morality, understood as an ineludible relation of the sets of rules denoted by such expressions.

With the third sentence, Green affirms that law and morals regulate the same ambit, the same "topic", and more precisely "high-stakes matters of social morality" (id., 1048). This thesis too, qua presumptive foundation of CT, is quite dubious. Though it may be envisaged that a legal system regulating the social life of a certain community cannot last for a long time without some sorts of correspondence with the positive morality of that community, it is not impossible to think of a legal system totally unconnected to the moral conception which is socially widespread.

[16] Normally, moral qualifications are mainly extensional, whereas legal qualifications (in particular criminal law ones) are eminently intensional. Indeed, morally forbidden actions are forbidden under any description, whereas legal forbidden actions are forbidden only under a specific characterization.

It must be noted that, in contemporary societies, characterized by a high degree of pluralism, legal systems assume, as it were, a "procedural nature", in that they institutionalize, in particular at the constitutional level, procedures aiming at conciliating, or balancing, different and conflicting social moral systems (or principles which are expressions of these systems). If this is correct, the provinces of law and morals are substantially different, since law would partially be a set of procedures to solve, on a meta-level, first order moral conflicts within a certain society.

The problems affecting this defense of CT are probably due to the overlooking of the fact that ST and CT, in their "substantial" versions, are liable to two different readings (provided by a set theory approach): one extensional and the other intensional. Once examined these readings, the problem of the relations of law and morals is significantly reduced (and, in some cases, even dissolved).

On the extensional reading, the relations of law and morals are, as it were, parasitic on single normative systems: one must see, case by case, what the connections (if any) are between some legal systems (at most, all the legal systems) and some moral systems (at most, all the legal systems). To hold a necessary connection between law and morals one should prove that there are elements that cannot but belong to such systems.

In the first place, one must note – with Bulygin (2007: 180) – that the connection thesis is ambiguous for it can express at least two different propositions: (i) there is only one moral system to which all legal systems are connected, or (ii) for any legal system, there exists a moral system to which it is connected[17].

The second proposition is not problematic at all, since it only affirms that legal systems are expression of a certain system of positive morality. In the terms of set theory, this means that the intersection of positive moralities and legal systems is not empty (viz. it's not an instance of the empty set). However, there is nothing necessary in such a connection: it is well possible that a legal system exists which is not the expression of the positive morality in force within a certain community or which is the expression of no moral system at all. On the set theory approach, this means that there is no necessary non-empty intersection of the set of legal rules and the set of moral rules in force within a certain society, although this intersection – as we have just seen – often is not empty *as a matter of fact*.

The first proposition states, instead, that all the legal systems conform to one ideal morality. But this thesis – beyond the quite controversial question of the very existence of an ideal morality and its cognoscibility[18] – is easily rebutted on the basis of logical analysis: (a) first, it is more than improbable (if not simply false) that all the legal systems, which are or have been in force, have the same *intersection* with ideal

[17] See also Rodríguez 2021: 152.

[18] Against moral objectivism and its criticisms of moral non-cognitivism, I argued extensively in Ratti 2021.

morality, unless one wishes to use the term 'legal system' to denote only those sets of rules which have a specific intersection with ideal morality, but here we are in the ambit of an intensional reading, having the nature of a stipulation (we will address this issue in the following pages); (b) second, one can hold that every legal system has at least one element (a rule whatsoever) contained in a non-empty intersection with ideal morality. Some authors – such as Garzón Valdés (1990: 111 ff.)[19] – maintain that such an element is the rule of recognition, i.e. the rule which provides the criteria of identification of the elements of the legal system, for it is accepted from the internal point of view at least by *officials*, who, by their acceptance, would express a claim of legitimacy of the legal system stemming from this rule. Both theses are ill-founded: the first because there is no conceptual warrant that any legal system must intersect, although minimally, with ideal morality; the second because – as Bulygin (2007: 181 ff.) asserts – one cannot infer the presence of legitimacy from the claim of legitimacy: on the set theory approach, this means that one cannot hold that any rule of recognition whatsoever is also a rule belonging (or logically derivable from a rule belonging) to ideal morality.

This is obvious, if the rule of recognition is regarded as a set of intensional criteria which allows one to identify a certain normative system, which would necessarily be connected to ideal morality if conformity (or some other connection) to it were established by such criteria, *in any empirical manifestation of them*. As a matter of course, this is tantamount to stipulating that there cannot be a rule of recognition (and, consequently, an act of identification of law) which is not somehow connected to morality: but this is question-begging, since it assumes what it should prove.

The most serious attempt to bind, from an *extensional* point of view, law and morals, is due, somewhat paradoxically, to one of the champions of XX century legal positivism: H.L.A. Hart (1994: 193 ff.) with his controversial thesis of the minimum content of natural law. The conjunction of the famous truisms (human vulnerability, approximate equality, limited altruism, limited resources, limited understanding and strength of will) and the assumption of survival as the aim "which men have in associating with each other" (Hart 1994: 193) leads to the view that "law and morals should include a specific content" (*ibid.*). Hart maintains that the connection between natural facts illustrated by the five truisms and the content of legal or moral rules is "distinctively rational". Hence, Hart's thesis may be rephrased (in non-Hartian terms) by saying that there is an anankastic relation between a certain end (survival) and the means to safeguard it (rules)[20]: on this reading, Hart's thesis would match the requirements of legal positivism, despite the misleading heading of "minimum content of natural law".

However, by this thesis, Hart wants to deny the Kelsenian tenet, according

[19] Cf. also Green 2008: 1048-1049.

[20] On the notion of an anankastic relation, see von Wright 1963: ch. 1.

to which law can have any content whatsoever[21]. Denying such a tenet means, of course, that law cannot have any content whatsoever. But this, in Hart's intent, does not mean that law cannot contain some elements; it rather means that the extension of legal systems *cannot but* contain some elements. Let us call these elements rules R1, R2, and R3: this means that any legal system should contain at least the following "hard kernel" in its extension: {R1, R2, R3}. Now, a normative system which does not include this hard kernel cannot be regarded, according to Hart's criteria, as a legal system. At the same time, Hart seems to suggest that moral systems too cannot but have an extensional "hard kernel", somehow analogous to law's kernel. For instance, rules which forbid physical aggressions are common to both kinds of normative systems. Let R1 be the name of the rule which prohibits physical aggressions. Thus, legal and moral systems would have, on this standpoint, a *necessary intersection*, for they *cannot but* contain R1. After all, law and morals would be *necessarily connected* from an extensional point of view.

There are at least two objections which can be raised to this view: (*a*) the first concerns the assumption of the end of survival (the conceptual pivotal point of Hart's argumentation): there's no reason why legal systems must necessarily serve the end of survival. The analysis of contemporary legal regulations about unrestrained consumption, economic development detrimental to environment or the workers' physical and mental integrity, free selling of firearms, seems to suggest otherwise[22]. (*b*) Second, one should note that, at the theoretical level, there is no reason why one should deny the qualification of 'legal' to a normative system which does not contain the rules which constitute the minimum content of natural law. Hart's definition is again begging the question, for it connects the presumed necessary extension of normative systems to the instrumental properties of some norms which compose them.

On the intensional reading, the relations between the two classes of normative systems under examination have to do with the very concepts of 'law' and 'morality'. More precisely, their relations depend on how such concepts *are defined*. As is known, stipulations of concepts are essentially arbitrary, so that it is always possible to forge a notion of 'law' which entails its connection with morality. In this sense, Bobbio's distinction between evaluative and non-evaluative definitions of 'law' keeps on being fundamental[23].

The latter – Bobbio (1996: 137) affirms – define "law understood as a system as a means serving an end: obviously, the end, in accordance to which law is defined,

[21] On this topic, see the seminal Paulson 1975.

[22] On this point see Green (2008: 1048), who correctly stresses that such a thesis is, actually, too strong: "Actually, unless 'survival' is understood in a vacuously broad way, Hart's claim is too bold: There are lots of suicide pacts around these days".

[23] Bobbio 1996: 137-143. Cf. also Hart 1983: 11-12.

varies from one philosopher to another". Immediately after, Bobbio adds: "one of the most traditional philosophical definitions is that which defines law in function of justice (i.e. as a system aiming at attaining justice)". It is clear that such definitions of the concept of law entail a necessary (*viz.* conceptual) connection of law and morals, for three reasons at least: (1) the criterion to determine the legality of the *set* of legal rules requires moral qualities characterizing such a set; and/or (2) the criterion to determine the membership of the rules to a legal system contemplates, from the very beginning, the presence of moral qualities in such rules; and/or (3) both the set and their single elements are to be regarded as legal in so far as they possess certain moral qualities. All this renders the question of the connection of law and morality absolutely trivial, since it is built on a stipulation, and hence becomes analytical.

It is more interesting to observe that some prominent defenders of contemporary positivism – such as Joseph Raz or Scott Shapiro – forge an *evaluative* concept of law, though intending, at the same time, to defend a kind of positivism that regards law as necessarily separate from morality.

As is known, Raz (1994) defends, *inter alia*, the normal justification thesis. Put very roughly, such a thesis affirms that, if law claims to have legitimate authority, those subject to legal rules are better off, as to what the correct and objective balancing of applicable reasons for action requires, by following the normative authority's directives rather than carrying out their personal balancing of reasons. This means that legal rules cannot incorporate moral criteria; otherwise, they could not carry out their guiding function towards the addressees of the rules, regarding what right reason requires.

In turn, Scott Shapiro (2001: 177 ff.) has defended the practical difference thesis as an essential element of exclusive legal positivism. Such a thesis affirms that a legal rule which included moral concepts would make no practical difference in agents' deliberation, since it would not block those axiological considerations which agents would carry out in the absence of such a rule and should avoid precisely in virtue of that rule. Consequently, an "inclusive" law would thwart law's main function of guiding human conduct by providing agents with genuine reasons for action.

In a more recent work, Shapiro (2011: 213 ff.) holds that law, unlike other normative systems, is characterized by the fact that its "mission is to address the moral defects of alternative forms of social ordering" (id., 213).

However, such definitions – supposedly positivistic, and hence non-evaluative – are neither substantially nor structurally different from the above-mentioned stipulations elaborated by natural law doctrine, widely considered as openly evaluative definitions. Indeed, all these definitions of 'law' – which share the feature of connoting the "object" law as a function of a certain value[24], be it justice, right reason, or its ability to furnish genuine reasons for action – lead fatally to a trivially necessary

[24] Bobbio 1996: 139.

connection of law and morality, created by stipulation. In particular, in Raz's and Shapiro's theses, one can find the paradoxical attempt of "demarcating" the elements which compose legal systems from those that compose moral systems on the basis of a strongly axiological reading of the function of legal systems, which ends up connecting them functionally to morality.

Overlooking this intensional feature of the relations between the concepts of law and morality – together with lack of an adequate distinction between sets and their elements – is, in my opinion, one of the biggest defects of the contemporary debate on legal positivism.

Is there any way out from this "impasse of intensionality"?

Once again, a way out is suggested by Bobbio (1996: 141): a non-evaluative definition of 'law' is merely factual, i.e. it connects the membership of rules to a legal system with empirical phenomena (such as, e.g., coercion or effectiveness), and not to their dependence on ends or values.

The subsequent problem is to identify the empirical resources to reach a definition of 'law' which is not completely arbitrary, i.e. which artificially separates or binds law and morals, or, more generally, law and non-law[25].

It suffices here to succinctly mention two of the most recent proposals of rational reconstructing the criteria of determination of the relevant facts to establish the legality of rules, formulated respectively by Brian Leiter (2007) and Mario Jori (2010).

According to Leiter (2007: 135), the ontological guide for the discovery of what "legally exists" must be a legal science elaborated in accordance with Quinean "replacement naturalism": what leads him to propose an "exclusivist" definition of what counts as a legal source, since the "best" legal science (both empirical and predictive of judicial sentences) identifies legal sources only in authority-based documents[26].

According to Jori (2010: 71) the ontological guide is rather common sense, which allows one to determine "which kinds of phenomena are identifiable as law in general and, amongst those phenomena, which one is identifiable as law in force in our society". This leads him towards a broader view than exclusive legal positivism, concerning the resources from which one can extract legal norms. Such a view holds that "legal sources can incorporate external elements of any sort and refer to ethical

[25] In the terms of set theory, ST could may be defined as the "thesis of complementation", in so far as it tries to determine the criteria to distinguish, within the universe of the normative, the complement of the legal (i.e. the non-legal).

[26] I am aware of the fact that Leiter (2011) has denied any possibility of successfully attempting to demarcate law and morals. The repercussion of this new thesis of Leiter on his project of naturalizing jurisprudence seems to me enormous, in the sense that I cannot see how Leiter can keep on holding that a naturalized theory of legal adjudication, presupposes an "exclusivist" thesis as to legal sources, such that it makes it possible to distinguish legal from non-legal sources. This is obviously impossible, once one rejects the possibility of demarcation, and so deprives naturalized jurisprudence of those which, for Leiter, are its conceptual bases. I have articulated this view in Ratti 2017.

and social opinion of any kind" (id., 107), and presupposes the previous "separation determined by a list of sources" (id., 108).

Though different, such definitions share the feature of being *non-evaluative*, by claiming to provide a notion of 'law' which captures, in a correct and empirically provable way, the criteria for the identification of the "object" law, actually used in our society.

In both cases, beyond the differences in the content, there is an induction of such criteria from social practice – legal science in Leiter, the general practice of rules' addressees in Jori. Unlike evaluative definitions, such definitions are not arbitrary: they are generalizations of 'definitions (explicitly or implicitly) in use' amongst legal agents or legal scientists.

One must conclude that the only fruitful definitions of 'law', in that they allow a profitable discussion of the relations of law and morals, are the non-evaluative ones, whereas evaluative definitions limit themselves to stipulating a notion of law that – as has been argued – connects or separates artificially, *in vitro*, both kinds of normative systems.

The difference in the criteria of Leiter and Jori shows how two different reconstructions of some empirical phenomena lead to two different views regarding the criteria of identification of legality: one having an exclusivist character, the other having a *lato sensu* inclusivist one[27]. There can be theoretical reasons to prefer one over the other: anyway, only deriving the intension of 'law' from provable social facts – what legal agents or jurists, in carrying out their activities, take as law – allows one to meaningfully discuss the question whether law and morality are connected or separate.

4. Setting Up the Frames

To identify the law in most regulatory states one must be able to identify not only a set of sources, but also their possible meanings. This, in turn, normally implies that one should be able to recognize a set of texts plus their meanings. From the point of view of set theory, this means, at a first approximation, that one must be able to spot a set of authoritative sentences connected, by interpretative sentences, to other sentences which are deemed synonymous to the original ones. Thus, we have at least three sets which are involved in the process of identification of law through interpretation. The set of authoritative texts, the set of interpretive sentences, and the set of norms, understood as the sentences deemed synonymous to the original authoritative texts. Clearly enough, there is no one-to-one correspondence among such sets. This thesis was famously articulated, though not in terms of sets, by Hans Kelsen (1992: 80-82) who puts forward the idea of legal interpretation as detection of a frame of possible meanings. Kelsen (1992: 80) writes:

[27] Cf. Jori 2010: 107-108.

> If "interpretation" is understood as discovering the meaning of the norm to be applied, its result can only be the discovery of the frame that the norm to be interpreted represents and, within this frame, the cognition of various possibilities for application.

And, elaborating on such a notion, Kelsen (*ibid.*) adds:

> That a judicial decision is based on a statute means in truth simply that the decision stays within the frame the statute represents, means simply that the decision is one of the individual norms possible within the frame of the general norm, not that it is the only individual norm possible.

This idea is clearly rephrased by Riccardo Guastini in terms of sets[28]. Guastini (2011a: 59) writes

> The set of possible meanings of a text – identifiable in the light of the rules of the language, the interpretative methods in use, the dogmatic theses widespread in legal dogmatics, etc. – can be called, after Kelsen: the "frame of meaning of the text".

The theoretical importance of the notion of a frame lies in the fact that it "is needed to classify the operations of the interpreters: in particular, to discriminate between proper interpretation and the creation of new law" (*ibid.*).

In fact, it is possible to discern between cognitive interpretation and adjudicative interpretation (and divide the latter into two variants) precisely on the basis of the notion of frame:

> (a) cognitive interpretation consists in identifying the frame, that is, in listing the possible (plausible) meanings of the text; (b) standard adjudicative interpretation consists in choosing one of the meanings included in the frame; (c) creative interpretation consists in attributing a meaning to the text that does not fit into the frame.

Guastini (*ibid.*) adds:

> the frame has vague, fluid outlines. It may happen, for example, that the norms N1 and N2 certainly fall within the frame, that the norm N4 certainly falls out of it, and that, however, the situation is uncertain as for the norm N3. So, it is undecidable whether N3 is to be ascribed to operations of mere interpretation or instead of normative creation. Nonetheless, at least for what concerns the norms N1, N2, and N4, distinguishing between interpretation and normative creation is possible and, I believe, highly significant.

[28] Guastini 2015 interestingly uses the language of sets to deal with the content of legal orders.

The expression "frame", as used by Guastini, denotes different sets and the product of different operations on sets which must be neatly distinguished.

A first set denoted by such a term consists of the meanings which can be potentially attributed to a certain regulatory text within a certain legal community. At this regard, Guastini (2011a: 61) states:

> by "interpretation" we must understood not any attribution whatsoever of meaning to the interpreted text, but an attribution of meaning that falls within the frame – of variable breadth – of admissible meanings.

He adds (2011a: 337)

> Cognitive interpretation (or "scientific interpretation" in Kelsen's terminology) consists in determining – by applying the different interpretative techniques in use – the different norms expressed, or the "frame" of meanings potentially expressed, by a normative provision.

At other times, the concept of frame has as its object "multiple pre-existing jurisprudential interpretations" (Guastini 2011a: 84). In this other sense, the frame is therefore the set of meanings *actually ascribed* by the courts, in the past, to a certain legal source.

Guastini (2011a: 60) correctly clarifies that

> a creative interpretation, which attributes to the text in question a different and further meaning than those considered possible in the cognitive interpretation (N4, let's say), has the effect of extending the frame of possible meanings. So, from that moment on, cognitive interpretation – a "good" cognitive interpretation – will have to recognize that new meaning (N4) as one of the admissible meanings.

From these considerations, it seems possible to derive at least three different notions of frame in terms of sets.

(1) A first frame is constituted by the set of legally admissible meanings, that is, the meanings considered acceptable in light of the interpretive techniques in use among jurists.
(2) A second frame consists, instead, of the set of meanings actually attributed to a certain text in the decision-making interpretation (especially in the jurisdictional sphere).
(3) Finally, a third frame is made of all those meanings that, although culturally inadmissible at a given moment t1, are conceivable as meanings potentially attributable in the future to a legal source by a competent entity or organ.

The notion of frame, however, can be used (and is sometimes used) to refer to the result of possible operations on the sets just mentioned.

It is possible to maintain that the term "frame" is sometimes understood, in a very broad sense, to be the union of the frames delineated in points (1)-(3), that is, all the (actually or potentially) admissible meanings, plus the meanings actually admitted.

More often, however, "frame" refers to the intersection or the union of the subclasses (1) and (2). In the first case (intersection), to be admissible, a certain meaning must belong both to the set of legally admissible meanings and to the set of meanings in force in the jurisdictional sphere. In the second case (union) it will suffice that it belongs to at least one of the two sets.

When the meanings actually attributed are believed to be, for that very reason, admissible (at least legally), the relevant frame will be determined only by the admissible meanings. This frame, furthermore, will be in a relation of inclusion with the frame of the meanings actually ascribed (which will therefore constitute a subclass of the admissible ones).

However, the opposite may also be the case. If it is accepted that, through creative interpretation, the list of admissible meanings is extended, then the set of meanings practiced will include the set of admissible meanings, improperly (if the two sets overlap) or properly (if the set of admitted meanings is, at least in a diachronic perspective, broader than that of admissible meanings).

Each time a meaning is chosen outside the frame, the complement of the set of admissible meanings is reduced, for it is evidently composed of those meanings considered inadmissible[29]. Indeed, whenever a (previously) inadmissible meaning is chosen by a court, it becomes legally admissible.

References

Alchourrón, C.E. (1987). *On the Philosophical Adequacy of Set Theories*, «Theoría - Segunda época», 5-6, 567-574.

Alchourrón, C.E., Bulygin, E. (1971). *Normative Systems*, Wien-New York, Springer.

Alchourrón, C.E., Bulygin, E (1979). *Sobre la existencia de las normas jurídicas*, Valencia (Venezuela), Universidad de Carabobo.

Alchourrón, C.E., Bulygin, E (1981). *The Expressive Conception of Norms*, in R. Hilpinen (ed.), *New Studies in Deontic Logic*, Dordrecht, Reidel.

Berto, F. (2007). *Logica da zero a Gödel*, Bari, Laterza.

Bobbio, N. (1996). *Il positivismo giuridico*, Torino, Giappichelli.

[29] A subclass, proper or improper depending on the case, of which it is constituted by the set of conceivable meanings, indicated above, at point (3).

Bulygin, E. (1995). *Norme, validità, sistemi normativi*, Torino, Giappichelli.

Caracciolo, R. (2012). *Acts, Normative Formulations, and Defeasible Norms*, in Ferrer Beltrán-Ratti (eds.), 2012.

Dauben, J.W. (1990). *Georg Cantor: His Mathematics and Philosophy of the Infinite*, Princeton (NJ), Princeton University Press.

Dworkin, R. (1986): *Law's Empire*, London, Fontana.

Ferrer Beltrán, J., Ratti, G.B., 2010. *Validity and Defeasibility in the Legal Domain*, «Law and Philosophy» 29, 601-626.

Ferrer Beltrán, J., Ratti, G.B. (2012a). *Theoretical Disagreements. A Restatement of Legal Positivism*, in D. Canale, G. Tuzet (eds.), *The Planning Theory of Law: A Critical Reading*, Dordrecht, Springer.

Ferrer Beltrán, J., Ratti, G.B. (2012b). *Defeasibility and Legality: A Survey*, in J. Ferrer Beltrán, G.B. Ratti (eds.), 2012.

Ferrer Beltrán, J., Ratti, G.B. (eds.), 2012. *The Logic of Legal Requirements. Essays on Defeasibility*, Oxford, Oxford University Press.

Ferrer Beltrán, J., Rodríguez, J.L. (2011). *Jerarquías normativas y dinámica de los sistemas jurídicos*, Madrid, Marcial Pons.

Gardner, J. (2001). *Legal positivism 5½ Myths*, «American Journal of Jurisprudence», 96, 198-228.

Green, L. (2008). *Positivism and the Inseparability of Law and Morals*, «New York Law Review», 83, 1035-1058.

Guastini, R. (2011a). *Interpretare e argomentare*, Milano, Giuffrè.

Guastini, R. (2011b). *La sintassi del diritto*, Torino, Giappichelli.

Guastini, R. (2015). *A Realistic View on Law and Legal Cognition*, «Revus», 27, 45-54.

Hart, H.L.A. (1983). *Essays in Jurisprudence and Philosophy*, Oxford, Clarendon Press.

Hart, H.L.A. (1994). *The Concept of Law*, Oxford, Oxford University Press, II ed.

Jori, M. (2010). *Del diritto inesistente*, Pisa, Edizioni ETS.

Kelsen, H. (1992). *Introduction to the Problems of Legal Theory* (1934), Oxford, Clarendon Press.

Lemmon, E.J. (1965). *Beginning Logic*, London, Nelson & Sons.

Leiter, B. (2007). *Naturalizing Jurisprudence. Essays on American Legal Realism and Naturalism in Legal Philosophy*, Oxford, Oxford University Press.

Leiter, B. (2011). *The Demarcation Problem in Jurisprudence: A New Case for Skepticism*, «Oxford Journal of Legal Studies», 31, 663-677.

Marmor, A. (2005). *Interpretation and Legal Theo*ry, Oxford, Hart, II ed.

MacCormick, D.N. (1983). *On Legal Decisions and Their Consequences: From Dewey to Dworkin*, «New York University Law Review», 58, 239-258.

Paulson, S. (1975). *Constraints on Legal Norms: Kelsen's View in the "Essays"* (reviewing *Hans Kelsen: Essays in Legal and Moral Philosophy* by Hans Kelsen, O. Weinberger, P. Heath), «University of Chicago Law Review», 42, 768-786.

Quine, W.V. (1955). *On Frege's Way Out*, «Mind», 64, 145-159.

Quine, W.V. (1960). *Word and Object*, Cambridge (Mass.), MIT Press.

Quine, W.V. (1961). *Mathematical Logic*, Cambridge (Mass.), Harvard University Press, rev. ed.

Quine, W.V. (1963). *Set Theory and Its Logic*, Cambridge (Mass.), Harvard University Press.

Quine, W.V. (1970). *Philosophy of Logic*, Englewood Cliffs (NJ), Prentice Hall.

Quine, W.V. (1974). *Methods of Logic*, London, Routledge & Kegan Paul, III ed.

Quine, W.V. (1980). *From a Logical Point of View*. Nine Logico-Philosophical Essays, Cambridge (Mass.), Harvard University Press.

Ratti, G.B. (2013). *El gobierno de las normas*, Madrid, Marcial Pons.

Ratti, G.B. (2017). *Naturalism and the Demarcation Problem*, in J. Stelmach, B. Brożek, Ł. Kurek (eds.), *The Province of Jurisprudence Naturalized*, Varsavia, Wolters Kluwer, 33-49.

Ratti, G.B. (2019a). *Cornici e insiemi*, in P. Chiassoni, P. Comanducci, G.B. Ratti (eds.), *L'arte della distinzione. Scritti per Riccardo Guastini*, vol. II, Madrid, Marcial Pons, 317-325.

Ratti, G.B. (2019b). *On Identifying the Law and Its Supposed Conventional Foundations. A Set-Theory Approach*, in L. Ramírez-Ludeña, J.M. Vilajosana (eds.), *Legal Conventionalism*, Dordrecht, Springer, 127-144.

Ratti, G.B. (2021). *Bertrand Russell and the Paradox of the Committed Emotivist*, «*Russell*: The Journal of Bertrand Russell Studies», 41, 5-31.

Raz, J. (1994). *Authority, Law, and Morality* (1985), in Id., *Ethics in the Public Domain*, Oxford, Clarendon Press.

Rodríguez, J.L. (2006). *La tensión entre dos concepciones del sistema jurídico*, «Análisis filosófico», 26, 242-276.

Rodríguez, J.L. (2021). *Teoría analítica del derecho*, Madrid, Marcial Pons.

Ross, A. (1958). *On Law and Justice*, London, Stevens & Sons.

Ross, A. (1988). *Validity and the Conflict between Positivism and Natural Law* (1961), in S. Paulson, B. Litchewski (eds.), *Normativity and norms*, Oxford, Oxford University Press.

Russell, B. (1918). *Introduction to Mathematical Logic*, Allen & Unwind.

Russell, B. (2010). *Principles of Mathematics* (1903, 1937), London, Routledge, II ed.

Shapiro, S.J. (2001). *On Hart's Way Out*, in J. Coleman (ed.), *Hart's Postscript*, Oxford, Oxford University Press.

Shapiro, S.J. (2011). *Legality*, Cambridge (Mass.), Harvard University Press.

Suppes, A. (1960). *Axiomatic Set Theory*, Princeton (NJ), Van Nostrand.

Von Wright, G.H. (1963). *Norm and Action. A Logical Inquiry*, London, Routledge & Kegan Paul.