Three Kinds of Logical Indeterminacy in the Law. Alf Ross's Insights

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Abstract

The paper analyzes Ross's theses about three kinds of logical indeterminacy which affect the law understood as a set of norms. First, the logical status of logically derived norms in the light of some undesirable consequences which follow from the application of deductive rules of inference to the normative domain (so-called Ross's paradox). Second, the indeterminacy of negation when applied to normative conditionals. Finally, the paper deals with the puzzle stemming from the application, to the rule on constitutional amendment, of the procedure that the same rule provides.

Keywords: Logical Indeterminacy. Logic of Norms. Ross's Paradox. Negation. Self-reference.

Foreword

In this work, I set myself a modest task: to analyze some theses defended by Alf Ross regarding the logical indeterminacy of law.

By "logical indeterminacy of law" I do not refer to the problems stemming from normative gaps and inconsistencies between norms. These flaws of legal systems are normally examined by means of logical tools. Their *origin*, however, is not logical, but rather empirical: a careful lawgiver may avoid them, to a great extent, by crafting a complete and consistent set of norms.

Instead, by "logical indeterminacy" I refer to flaws which stem from logical features of normative discourse which not even the most careful of the lawgivers could completely escape. They are, so to speak, conceptual, rather than empirical drawbacks.

Not surprisingly, Alf Ross's analysis is, on this score, pioneering. In the early for-

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ties of the past century he discovered a paradox that still bears his name and haunts deontic logic and legal reasoning alike; in the late sixties he deepened our mastery of the notion of a conditional norm and its negation and was so able to dismiss the too hasty analogy which is frequently made between descriptive conditionals and normative conditionals, and in a period spanning more than a decade (1958, 1969) he discovered a foundational self-referential puzzle which jeopardizes the very idea of legal determinacy.

I shall analyze these three issues in this order.

1. Ross's Paradox and the Notion of a Logical Consequence in the Normative Domain

The logical riddle known as "Ross's paradox" is easily explained[1].

If rules of inference of standard propositional logic are applied to the normative domain, we obtain that by applying the following tautological sentence[2]

[1] $$A \rightarrow AvB$$

to the realm of norms, we get

[2] $$OA \rightarrow O(AvB)$$

This is so because the truth of A entails the truth of AvB, since the latter is true whenever at least one of its components is true. So, if A is indeed true, then AvB is unavoidably true as well (because A is true).

Transferred to the normative domain, we oddly obtain that from an obligation to do A (or to create the state of affairs A), we can infer an obligation to do AvB, that is an alternative obligation, according to which we can freely choose to create the state of affairs A and/or the state of affairs B. In Ross's example, the norm "Mail the letter!" entails the norm "Mail the letter or burn it!"

Commonly, this inference is characterized as an uneasy one, since it seems to convey a freedom of choice for the norm-addressee that was not present in the issuance of the categorical norm[3].

Another version of the paradox – propitiated by the equivalence between AvB and ¬A→B – is the following:

[3] $$OA \rightarrow O(\neg A \rightarrow B)$$

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1 For recent discussion, see Guastini 2015.

2 The symbol "¬" represents negation, "&" is for conjunction, "v" for disjunction, and "→" for the material conditional.

3 See von Wright 1991: 281.

This means that OA implies that "If not-A occurs than everything is obligatory" and sounds even more paradoxical than the original formulation. To keep on with Ross's example, from "Mail the letter!" it follows that "If you don't mail the letter, then burn it!".

This paradox is groundbreaking in its simplicity. Indeed, if Ross is right, the logical behavior of norms, and thus also of legal norms, is just an illusion or the product of self-deception. The consequences of the application of logic to law would be so absurd that no one would accept it as rational.

Here I shall analyze three possible arguments used to rebut Ross's paradox[4]. As we shall see, such arguments are unconvincing or not wholly persuasive.

The first argument can be dubbed the *argument from equivalence* and is found in Castañeda (1981: 65) and, more recently, in Woleński (2016: 121). Let us consider the formulation of the point provided by the latter:

> When Alf Ross invented the paradox captured by the formula OA → O(AvB), it was considered to have plagued the logic of normative discourse. [...] because B in Ross's paradox is arbitrary, we can have either OA → O(AvB) or OA → O(Av¬B). Now, given that formula A is equivalent to the conjunction (AvB) & (Av¬B), we have a simple argument that Ross's paradox is apparent.

This argument is not conclusive, I submit, since nobody denies that, from a logical point of view, OA is equivalent to O(AvB)&(Av¬B). An example: that it is categorically obligatory to pay taxes implies that, in both worlds "B" (say, sunny days) and "¬B" (non-sunny days) one ought to pay taxes: what is aptly captured by the formula O(AvB)&(Av¬B). But Ross's paradox does not merely bear on the formulation of categorical norms. It states that, given any categorical norm, such a norm implies another norm whose content is the original one in disjunction with any other content. To avoid such an implication it does not suffice to say that A is equivalent to the conjunction (AvB)&(Av¬B). The fact that it is not logically paradoxical that from OA we can derive both O(AvB) and O(Av¬B) (and thus O(AvB)&(Av¬B)) does not deny that, from a normative stance, it is indeed paradoxical that, from a categorical norm, we can derive an alternative obligation (and, considered that OA → PA is regarded as valid in standard systems of logic of norms, also a disjunctive permission). If the guidance of human conduct is the aim of normative discourses (law included), there is something quite odd in allowing such an inference. This is the gist of Ross's paradox: to apply deductive logic to norms brings about irrational consequences. One cannot use tautologies of deductive logic to show that it is not true, for they only show that Ross's derivations are correct and so the paradox persists.

[4] On several other arguments against Ross's paradox, see Alarcón Cabrera 1990.

The second argument, which is partially similar to the one we have just analyzed, can be dubbed the *argument from satisfaction* and is found in von Wright (1991: 281).

Elaborating on Ross's example, which we previously mentioned, von Wright observes that mailing and burning the letter at the same time is impossible. So that in O(AvB)&O(Av¬B) – which, as we know, is equivalent to OA – the first conjunct is not possible. As a consequence, the only way of satisfying (viz. complying with) both norms OA and O(AvB) is by doing A and ¬B. Von Wright is of course right, but, as he manifestly admits, this only "mitigates" the paradox, but does not solve it. Indeed, the example is just a particular application of the paradox in a situation where both disjuncts are exclusive: the paradox is apparently diminished only by this very fact. Indeed, it is not even a real mitigation: it is just a case where a property of a proposition implies the negation of the other. But it must be clear that in a case where the disjuncts are not exclusive, the paradox is no way mitigated. Let us suppose that from the command "Mail the letter!" we derive "Mail the letter or have a shower!" Clearly, now we have no reason to say that the only way of satisfying the original norm plus the derived one is mailing the letter and not having a shower. The paradox still stands.

The third argument, which can be named the *argument from entailment* is also found in von Wright (1991: 281-2) and, as it were, "got the grain right". Von Wright observes that the paradox has to do with the notion of a logical consequence. That is to say: what does it mean that a certain norm *logically follows* from another norm? Von Wright emphatically denies that "entailment" denotes a *genetic relation*: it is not the case that if a certain norm OA exists (has been given by the lawmaker), then the norm O(AvB) also exists (has been given, at least implicitly, by the lawmaker)[5].

According to von Wright, differently from what occurs in propositional logic, the concept of entailment is conceptually reducible to that of consistency. Indeed, the concept of consistency works here as a primitive one: a norm N1 entails another norm N2 when the norm-negation of N2 (i.e. the norm which makes undoable the simultaneous conjunction of the contents of both norms) is incompatible with N1. A simple example may help: OA entails PA, since the negation-norm of the latter (i.e. O¬A) is incompatible with OA, since A&¬A is not a doable state of affairs.

As a consequence, "That an obligation to mail the letter entails an obligation to mail or burn it "only" means that the first is [...] incompatible with a permission to leave the letter unmailed and do something else" (id., 281). Von Wright's reasoning may be reformulated as follows: indeed, OA and P(¬A&¬B) are inconsistent, since A&¬A&¬B is a contradictory proposition as far as logic is concerned. So, the norm-negation of the latter norm – which is O(AvB)[6] – is entailed by the former (viz. OA).

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[5] On this point, see Kelsen 1973: 228-253.

[6] P(¬A &¬B) is negated by ¬P(¬A &¬B), which in turn is equivalent to O¬(¬A &¬B). This, in turn, is equivalent to O(AvB).

As is easy to see, here we have a complete reduction of entailment to the notion of consistency: indeed, if we do not know what the relations of consistency between norms are, we cannot determine whether a norm entails another norm. But this is very different from propositional logic, where all genuine (viz. non-trivial) entailments presuppose consistency, but not any consistency presupposes an entailment.

Accordingly, von Wright's interesting argument cuts no ice against Ross's paradox, since the gist of such a paradox is exactly that the application of propositional logic to norms brings about undesirable results. However, von Wright's solution (aiming to eliminate such results) is predicated on the *equivalence* of consistency and entailment. But this is not a law of propositional logic, as Quine (1959: 29, 34) clarified. Von Wright is using propositional logic to determine the rationality of a lawgiver (where "rationality" means not issuing conflicting norms), but not to determine which norms follow from other norms. In this sense, von Wright's argument leaves Ross's paradox unchallenged.

2. Normative Conditionals and their Negations

Very few things may be said to be more agreed upon amongst jurists than the thesis that legal norms are, by and large, to be regarded as conditionals connecting some key operative facts to legal consequences. When it comes to applying logic to such an intuition, jurists usually refer to very common schemata of reasoning (such as *modus ponens*) to support the view that legal conditionals, in many ways, have the same logical behavior as material conditionals have in propositional logic.

However, Ross highlighted that, at some junctures, normative conditionals have specific features that characterize them in opposition to descriptive material conditionals. In particular, Ross (1968: 173-4) clarifies that the joint issuance of two conflicting conditional norms can be reconstructed at least in two forms: the first by using external negation: $O(A \rightarrow B) \& \neg O(A \rightarrow B)$; the second using internal negation: $O(A \rightarrow B) \& O(A \rightarrow \neg B)$.

The former, which is the favorite way of reconstructing conflicting conditional obligations among deontic logicians, represents the conflict between the obligation of making true the state of affairs $A \rightarrow B$ and the permission of making true the state of affairs $A \& \neg B$, which is the negation of $A \rightarrow B$. According to Ross (id., 174), the conjunction $O(A \rightarrow B) \& \neg O(A \rightarrow B)$ is *pure nonsense*, for it does not prescribe anything at all.

The latter formula (viz. $O(A \rightarrow B) \& O(A \rightarrow \neg B)$) is intuitively closer to what lawyers regard as a normative conflict, but has an important drawback. Indeed, as Ross correctly observes, from the stance of propositional logic, the sentence providing that $O(A \rightarrow B) \& O(A \rightarrow \neg B)$ is not an inconsistency, since it does not imply any sentence whatsoever. It is far from being pure nonsense. Rather, it logically entails

O¬A[7]. This means that a lawgiver issuing the conjunction under exam is far from being irrational. He can be, as it were, a "malicious" lawgiver, but totally rational (indeed, he is *logically rational*). In issuing such a conjunction of norms, the lawgiver may be regarded as entailing the norm according to which the addressee should not put himself in the scenario that triggers the application of the conflicting consequences (id., 174).

Ross is not clear whether legal conditional norms are to be represented, regarding their negation, as denied conditionals (viz. by using external negation) or conditional denials (viz. by using internal negation).

As we have seen, the simultaneous membership of O(A→B) and ¬O(A→B) within a certain normative system brings about a normative inconsistency, but this does not seem a correct reconstruction of actual antinomies between *conditional norms* which are found in normative documents and the way they are read by jurists. Indeed, the norm-negation of O(A→B) – that is, ¬O(A→B), amounting in turn to P(A&¬B) – is logically well-formed, but seems to lose any feature of conditionality. In fact, it amounts to a categorical norm permitting to bring about simultaneously the states of affairs A and ¬B.

Norms formed by means of internal negation are better in representing what jurists have in mind when dealing with normative conflicts: the same case connected to incompatible normative consequences. In symbols, as we know: O(A→B)&O(A→¬B). However, as we discussed, from a logical point of view, far from bringing to the trivialization of the case A, the conjunction at hand entails the obligation of not-A. This sounds paradoxical regarding many cases. In particular, in all those cases where the common antecedent is not liable to be controlled by the norm-addressee. Consider a norm imposing mandatory military service to young males when turning eighteen and another norm permitting conscientious objections to military service to young males when turning eighteen. Of course, from the conflict of these two norms, nobody would derive the absurd consequence that now there is an obligation for young males not to turn eighteen.

Another candidate for symbolizing a normative conditional could be A→OB, but according to Ross (1968: 167), this is an "impossible hybrid", for it mixes descriptive aspects with prescriptive ones and symbolizes something which is

[7] Analogously with what happens in propositional logic. This is easily seen from the following truth-table:

A→B	&	A→¬B	≡	¬B
1 1 1	*0*	1 0 0	**1**	*0*
1 0 0	*0*	1 1 1	**1**	*0*
0 1 1	*1*	0 1 0	**1**	*1*
0 1 0	*1*	0 1 1	**1**	*1*

neither indicative nor directive[8].

Here we face a huge problem of logical indeterminacy: legal conditionals cannot be easily accommodated into a univocal logical form. This can be seen in particular as regards the negation of conditional norms. Since it is not clear whether the negation found in ordinary prescriptive discourses must be read as internal or external, and whether the simultaneous conflict of a norm and its negation brings about the inconsistency of the normative system or not, it is quite clear that law (as any other system of conditional norms) suffers from this specific kind of indeterminacy. Moreover, sometimes the reconstruction in terms of internal negation brings about paradoxical results. As a consequence, the widespread thesis that legal norms are, generally, conditional norms may be read in different ways, based on the notions of conditional and consistency we apply. However, it must be clear that the law is logically indeterminate in so far as we decide which notions we wish to apply and, in some cases, even after this choice has been made.

3. The Problem of Self-Reference and a Puzzle for Constitutional Revision

In several works, Alf Ross held that problems of self-reference are paramount in legal systems. Particularly, in Ross (1958: 78-84; 1969), one can find a substantial treatment of the question of self-reference in legal provisions bearing on constitutional amendment.

In a nutshell, the problem is the following[9]: may the legal procedure for amending the constitution be used to modify the norm which provides the legal procedure for amending the constitution?

In literature, two main answers to this question are found.

(i) The first, negative, answer is usually maintained by pointing to two remarkable circumstances: (a) the self-referential character which affects the rule on constitutional amendment; (b) the fact that modifying a norm on the basis of what such a norm provides seems to lead to an inevitable contradiction.

Ross (1969: 7-17), holds, on the one hand, that (a) self-referential sentences are always meaningless. Such a contentious view has now been almost entirely rejected, on the basis that some self-referential statements seem to be perfectly sound, and paradoxes usually connected to self-reference may also rise within a set of non-self-referential statements[10].

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[8] Similarly, von Wright 1983: 151 regarding formulas as A→OB states "that if the standard connective in question is a truth-connective, then this 'linguistic hybrid' is a monster with no place in meaningful discourse'.

[9] For the following explanation, I am indebted to Guastini 2010: 229-231; and Bulygin 2015: 178-183. See also Moreso 1991.

[10] See Yablo 1993.

On the other hand, Ross also affirms that (b) there is a contradiction affecting the following reasoning concerning constitutional amendment:

> [1] N': The constitution may be amended by a process in accordance with conditions Q, and only by this process;
> [2] N" (stating that the constitution may be amended by a process, in accordance with conditions R) has been created in accordance with conditions Q;
> [3] Hence, N" is valid, that is, the constitution may be amended by a process in accordance with conditions R and only by this process.

As a matter of course, [1] and [3] appear as *prima facie* contradictory and, if it is really so, the reasoning at hand is not sound: at least one of its premises, in fact, must be false.

(ii) The second, positive, answer refers to the interaction of time and validity. Along these lines, Bulygin (2015: 182-183) holds that both norms – N' and N" – belong to different, so to speak, "momentary constitutions": say, C1 and C2. Therefore, even if a real contradiction existed, it would be between two norms belonging to different sets of norms. However, since "constitution" in N' refers to C1, whereas the same expression in N" refers to C2, there is no real contradiction in the piece of reasoning under examination.

Now, some remarks are in order here. The second answer seems to assume what it has to prove: *i.e.*, that N" is indeed *substituting* N'. This is true only in two cases: (i) if the legal order provides the chronological criterion for solving inconsistencies; (ii) if N" contains an expressed derogatory clause bearing on N'. In Ross's example this is not mentioned, and on this basis one can safely assume that [1] and [3] are indeed contradictory, for both norms seem to belong to a certain momentary constitution C2, which follow to the promulgation of N" at time t2 (whereas C1, which was in force at time t1, was, by hypothesis, formed by N' only). After all, then, Ross seems to be right on this score.

However, apart from this controversial point, neither of these answers seems to be entirely satisfactory. Let me elaborate. As Rodríguez (2006: 247) has pointed out, the criterion of *legality* is better seen as applying to *sets* of legal norms, rather than to single legal norms. As is known, the criterion of legality, traditionally understood, provides that rules produced by a legally competent authority are valid in a certain legal system until they are made invalid in the way determined by the legal system itself. According to Rodríguez, the principle of legality must be reinterpreted as stating the conditions to be met for a legal set (what may be called a "momentary legal system") to belong to a sequence of legal sets (which we can stipulate to call a "legal order"). It does *not* state the conditions that *a single norm* must meet to belong to a certain legal order.

Now, if this is correct, the amendment of the rule on amendment would be tantamount to breaking the continuity of a set of legal sets [*i.e.* a legal order], precisely when the rule on amendment is not regarded as self-referential: it would be a breach of legality, an illegitimate change in the legal order, since there is no rule which allows the amendment of the rule on amendment. This is due to the fact, correctly emphasized by Ross, that the apical normative authority of a certain legal order is that which can amend the constitution. But whereas Ross is wrong in dismissing all kinds of self-reference as nonsense, he is right in suggesting that the only way of making sense of a change of an apical rule of constitutional amendment is by referring to an (hypothetical and extralegal) basic norm, imposing to identify valid law by applying each subsequent norm on the amendment of momentary legal systems. By removing such a fiction, however, one obtains that the legal order (not only momentary legal systems) changes whenever the rule on constitutional amendment is modified.

However, if the rule on constitutional amendment is regarded as self-referential, a different problem arises: such a norm in fact carries out two different delegations to two conceptually different normative authorities (although the same organ or set of organs may be, empirically, vested by both delegations). In the case of the revision of the other norms of the constitution, in fact, it simply empowers some delegated authorities to amend such provisions. Nonetheless, regarding itself, it delegates a certain normative authority to change the criterion of membership of momentary legal systems to the legal order: it changes the legal order.

Accordingly, the determination of the self-referential character of provisions on constitutional amendment seems to lead to a puzzling dilemma regarding law's determinacy: either self-reference is banned but the legal order, as we have just seen, changes whenever an amendment of the rule on amendment is carried out, or self-reference is accepted, but then the apical normative authority of a legal order recognizes that it is not the apical normative authority of such an order, since it empowers another normative authority to modify the constitution and, consequently, the legal order. As a consequence, Ross's puzzle, even if read from a diachronic stance on law, persists.

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